Discussion of “Bridging the gap between theory and practice in basic statistical process monitoring”

Rob Goedhart, Marit Schoonhoven, Inez M. Zwetsloot & Ronald J. M. M. Does

To cite this article: Rob Goedhart, Marit Schoonhoven, Inez M. Zwetsloot & Ronald J. M. M. Does (2017) Discussion of “Bridging the gap between theory and practice in basic statistical process monitoring”, Quality Engineering, 29:1, 18-21, DOI: 10.1080/08982112.2016.1210453

To link to this article: http://dx.doi.org/10.1080/08982112.2016.1210453
Discussion of “Bridging the gap between theory and practice in basic statistical process monitoring”

Rob Goedhart, Marit Schoonhoven, Inez M. Zwetsloot, and Ronald J. M. M. Does

Department of Operations Management, Amsterdam Business School, University of Amsterdam, Amsterdam, The Netherlands

In his comprehensive review on statistical process monitoring (SPM), Professor William H. Woodall (2017) highlights several gaps between theory and practice. He discusses the need for more attention to Phase I data analysis, the use of the range to estimate the process standard deviation, the importance of modeling in SPM, the increased attention to profile monitoring, the effect of the estimation error, and ways to narrow the gap between theory and practice.

We agree with Professor Woodall on most of the raised issues: that the use of control charts and other monitoring methods should be referred to as SPM, on the usefulness of theory and probability distributions, on the importance of making SPM methods easy to use, and on the need to write articles with practitioners in mind.

In this discussion we share our thoughts on two topics. In the first place, we will discuss the choice of the estimation method for the standard deviation in Phase I. Secondly, we focus on dealing with the effect of estimation error on the performance of control charts and discuss alternative design methods. To conclude we provide some final thoughts. Furthermore, we wish to thank Professor Woodall for his thoughtful and interesting article and presentation during the Fourth Stu Hunter Research Conference in Waterloo, Canada.

Estimation methods for standard deviation in SPM

Woodall (2017) argued against the use of range based estimators for process dispersion in Phase I. He argues that, although the range is easy to use for practitioners, its relative efficiency (based on the mean squared error) compared to estimators based on the sample standard deviation is far behind. We agree with this view. In this section we discuss some alternative estimators.

The offered alternative in Woodall (2017) is based on the sample standard deviation, which is a very efficient estimator. However, when choosing an estimator it is important to realize that in practice Phase I data often contain contaminated observations. These contaminations are problematic as they can influence the parameter estimates, which negatively affects the performance of control charts in Phase II. Therefore, we advocate the use of alternative, more robust rather than only highly efficient, estimation methods.

In order to produce parameter estimates, a Phase I dataset is required. For our demonstration we consider Phase I data, consisting of 50 samples of 5 observations each, to be independent and normally distributed, $N(\mu + \delta \sigma, \tau \sigma^2)$. When $\delta = 0$ and $\tau = 1$, the process is considered in-control; otherwise the process has changed. To model data contaminations, 5% of the data comes from a shifted distribution, either diffuse variance disturbances ($\tau \neq 1$) or diffuse mean disturbances ($\delta \neq 0$); see Schoonhoven and Does (2012, 2013) for details and additional contamination scenarios.

We consider the efficiency of six standard deviation estimators, namely the mean of the sample standard deviations ($\bar{S}$), the mean of the sample ranges ($\bar{R}$), the mean of the sample interquartile ranges ($IQR$), the 20% trimmed mean of the sample interquartile ranges ($IQR_{20}$), a robust estimator ($D7$) proposed by Tatum (1997), and the adaptively trimmed standard deviation ($AT\bar{S}$). We illustrate the performance of these estimators for the uncontaminated case, i.e., the situation...
where all the data are \(N(\mu, \sigma^2)\) distributed, as well as two types of disturbances.

For each of the above standard deviation estimators and under each of the three scenarios, the degree of efficiency is assessed by determining the mean squared error (MSE). A lower MSE is desirable. The results can be found in Figure 1. The vertical axis of each graph in Figure 1 reflects the precision of the estimator (MSE) and the horizontal axis the magnitude of the contaminations.

As can be observed, the standard estimators \(\bar{S}\) and \(\bar{R}\) are not robust. The \(IQR\) is less efficient under normality when there are no contaminations, but performs reasonably well when there are diffuse variance disturbances. The \(IQR_{20}\) performs reasonably well for all types of contaminations but its efficiency is relatively low under normality. \(D7\) is efficient under normality, but has varying performance for disturbances.

These results show that no single robust point estimator performs well in all considered situations. An alternative to point estimators is to first screen the data. A Phase I control chart can be used for this purpose. In order to ensure robust Phase I limits an estimator should be used that is robust against different types of contaminations, for example \(IQR_{20}\). Then the Phase I data should be screened for contaminations. Based on the cleaned data the standard deviation can finally be estimated and, in order to ensure an efficient estimate, \(\bar{S}\) can be used. The procedure is the basis for the estimator \(ATS\) in Figure 1. We can see that this estimator is rather efficient under normality and more robust to contaminations than the point estimators. This procedure for the Shewhart control charts is worked out in Schoonhoven and Does (2012, 2013).

In summary, we agree with Woodall (2017) that the use of range-based estimation methods is not optimal. However, the alternative of efficient estimators based on samples standard deviations is only an option when the Phase I data are in-control. As soon as contaminations may be present, which is quite often the case in practice, these estimation methods break down. Hence, other alternatives, for example based on screening and robust estimators, can be used to decrease the gap between theory and practice.

### Estimation uncertainty and adjustments

Another issue raised by Woodall (2017) is the extent of the effect of estimation error. He convincingly shows that this effect is quite large; control charts based on estimated parameters show strongly varying performance. This effect is especially large for small sample sizes in Phase I (cf. Quesenberry, 1993). Woodall (2017) advocated that the design of control charts should be based on conditional performance instead of marginal performance, as this is “a reasonable approach to control the percentage of in-control ARLs that are below a given value”. We agree with the usefulness of conditional design of control charts, and would like to offer our view on possible directions to design these charts.

The initial attempts in the literature to design charts that take estimation error into account are based on marginal performance. Albers and Kallenberg (2005) derive correction factors for Shewhart control chart limits such that the expected ARL equals a pre-specified value. Their corrections have been improved by Goedhart et al. (2016a). However, this approach
generates a good control chart performance in expectation. Due to the large variation in in-control ARL values, a large proportion of the practitioners will then still have a bad control chart performance.

An alternative design criterion is to guarantee a minimum in-control ARL that a practitioner will achieve with a predefined probability. Gandy and Kvaløy (2013) designed a bootstrap procedure to derive control limits that guarantee this conditional performance, which has been simplified by Saleh et al. (2015). Having the gap between theory and practice in mind, we should realize that the mathematical complexity of the bootstrap based procedures requires an advanced level of statistical knowledge. Moreover, it is difficult to explain to users that every time one designs a control chart, one gets different control limits (due to bootstrapping). To overcome this complexity issue we look towards possible alternative procedures based on analytical approximations.

Next to the corrections for the marginal performance, Albers and Kallenberg (2005) also derive correction factors for the conditional performance criterion discussed above. However, the performance of these corrections is not yet as desired. Therefore, Goedhart et al. (2016b) derived new corrections factors to guarantee a desired conditional performance with a specified probability. To illustrate the implications of such a minimum performance guarantee, we simulate the distribution of the ARL of 1,000 estimated control charts, based on Phase I datasets consisting of 50 samples of size 5 each. The control charts are designed to guarantee a minimum in-control ARL of 370 with 90% probability. The results are shown in Figure 2; the boxplots indicate the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles. As can be observed in Figure 2, the 10th percentile is indeed located at 370 for both the bootstrap method of Gandy and Kvaløy (2013) and the approximations of Goedhart et al. (2016b); the differences between the bootstrap and the analytical approximations are negligible. We can also see that the factor of the unadjusted Shewhart control chart and the factors of Albers and Kallenberg (2005) lead to more estimated control charts with an ARL lower than 370.

Final thoughts

Woodall (2017) concludes with ideas on how to improve SPM research and practice. We would like to add an additional idea to point 9. Here, Woodall (2017) states that the assumption of known in-control parameter values is an exceptionally strong one. We would like to add to this point that alternatively assuming in-control data in Phase I is an exceptionally strong one as well. In our experience, most Phase I datasets are not perfectly in-control. In danger of overselling our own methods—one of Woodall’s warnings—we recommend the use of robust estimation methods in Phase I.

Furthermore, considering the effect of estimation error, we agree with Woodall (2017) that the conditional design is a promising approach to deal with the adverse effects of estimation error. Recently, bootstrap methods have been proposed to implement this design.
Though these provide an accurate control limit it is a very complex and mathematically advanced technique. In order to make this approach more accessible we offer an alternative solution, namely the use of analytical approximations. Such approximations can be used to determine analytical expressions for the required control limits. Although still complex, these approximations can easily be implemented in software, with all the complexity nicely hidden for the user.

**About the authors**

Rob Goedhart obtained his M.Sc. in econometrics in 2014 at the University of Amsterdam, the Netherlands. He is a Ph.D. student in the Department of Operations Management and consultant at the Institute for Business and Industrial Statistics of the University of Amsterdam, the Netherlands. His current research topic is control charting techniques with estimated parameters.

Marit Schoonhoven obtained her Ph.D. in statistics in 2011 at the University of Amsterdam, the Netherlands. She is an associate professor at the Department of Operations Management and senior consultant at the Institute for Business and Industrial Statistics of the University of Amsterdam, the Netherlands. Her current research interests include control charting techniques and operational management methods.

Inez M. Zwetsloot obtained her Ph.D. in statistics in 2016 at the University of Amsterdam, the Netherlands. She is an assistant professor in the Department of Operations Management and consultant at the Institute for Business and Industrial Statistics of the University of Amsterdam, the Netherlands. Her current research interests include robust exponentially weighted moving average charts and statistical process monitoring.

Ronald J. M. M. Does is Professor of Industrial Statistics at the University of Amsterdam, Director of the Institute for Business and Industrial Statistics, Head of the Department of Operations Management, and Director of the Institute of Executive Programmes at the Amsterdam Business School. He is Fellow of the ASQ and ASA, and Academician of the International Academy for Quality. His current research activities include the design of control charts for nonstandard situations, healthcare engineering, and operational management methods.

**References**


