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ABSTRACT Cumulative sum (CUSUM) control charts are very effective in detecting special causes. In general, the underlying distribution is supposed to be normal. In designing a CUSUM chart, it is important to know how the chart will respond to disturbances of normality. The focus of this article is to control the location parameter using a CUSUM structure and the major concern is to identify the CUSUM control charts that are of more practical value under different normal, non-normal, contaminated normal, and special cause contaminated parent scenarios. In this study, we propose and compare the performance of different CUSUM control charts for phase II monitoring of location, based on mean, median, midrange, Hodges-Lehmann, and trimean statistics. The average run length is used as the performance measure of the CUSUM control charts.

KEYWORDS average run length (ARL), contaminated environments, control charts, CUSUM statistic, fast initial response, normality, process location

INTRODUCTION

Statistical process control is a collection of different techniques that help differentiate between the common cause and the special cause variations in the response of a quality characteristic of interest in a process. Out of these techniques, the control chart is the most important one. It is used to monitor the parameters of a process, such as its location and spread. Some major classifications of the control charts are variable and attributes charts, univariate and multivariate charts, and memoryless charts (Shewhart's type) and memory charts (like cumulative sum [CUSUM] and exponentially weighted moving range [EWMA]). The commonly used Shewhart's variable control charts are the mean ($\overline{X}$), median, and mid-range charts for monitoring the process location and the range (R), standard deviation (S), and variance (S^2) charts for monitoring the process variability (cf. Montgomery 2009). The main deficiency of Shewhart-type control charts is that they are less sensitive to small and moderate shifts in the process parameter(s).

Another approach to address the detection of small shifts is to use memory control charts. CUSUM control charts proposed by Page (1954) and EWMA control charts proposed by Roberts (1959) are two commonly used memory-type control charts. These charts are designed such that they use the past information along with the current information, which makes them very sensitive to small and moderate shifts in the process parameters.

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In order to obtain efficient control limits for phase II monitoring, it is generally assumed that the process has a stable behavior when the data are collected during phase I (cf. Vining 2009; Woodall and Montgomery 1999). Most of the evaluations of existing control charts depend on the assumptions of normality, no contaminations, no outliers, and no measurement errors in phase I for the quality characteristic of interest. In case of violation of these assumptions, the design structures of the charts lose their performance ability and hence are of less practical use. There are many practical situations where non-normality is more common (see, for example, Janacek and Miekle 1997). One of the solutions to deal with this is to use control charts that are robust against violations of the basic assumptions, like normality.

To date, numerous papers on robust control charts have been published. Langenberg and Iglewicz (1986) suggested using the trimmed mean $\overline{X}$ and R charts. Rocke (1992) proposed the plotting of $\overline{X}$ and R charts with limits determined by the mean of the subgroup interquartile ranges and showed that this method resulted in easier detection of outliers and greater sensitivity to other forms of out-of-control behavior when outliers are present. Tatum (1997) suggested an interesting method for robust estimation of the process standard deviation for control charts. Moustafa and Mokhtar (1999) proposed a robust control chart for location that uses the Hodges-Lehmann and the Shamos-Bickel-Lehmann estimators as estimates of location and scale parameters, respectively. Wu et al. (2002) studied the median absolute deviations–based estimators and their application to the $\overline{X}$ chart. Moustafa (2009) modified the Shewhart chart by introducing the median as a robust estimator for location and absolute deviations to median as a robust estimator for dispersion. Recently, properties and effects of violations of ideal assumptions (e.g., normality, outliers free environment, no special causes, etc.) on the control charts have been studied in detail by Riaz (2008) and Schoonhoven et al. (2011a, 2011b).

Some authors have also discussed the robustness of the CUSUM chart to situations where the underlying assumptions are not fulfilled. Lucas and Crosier (1982) studied the robustness of the standard CUSUM chart and proposed four methods to reduce the effect of outliers on the average run length (ARL) performance. McDonald (1990) proposed the use of the CUSUM chart that is based on a nonparametric statistic. He used the idea of ranking the observations first and then using those ranks in the CUSUM structure. Hawkins (1993) proposed a robust CUSUM chart for individual observations based on Winsorization. MacEachern et al. (2007) proposed a robust CUSUM chart based on the likelihood of the variate and named their newly proposed chart the RLCUSUM chart. Li et al. (2010) proposed a nonparametric CUSUM chart based on the Wilcoxon rank-sum test. Reynolds and Stoumbos (2010) considered the robustness of the CUSUM chart for monitoring the process location and dispersion simultaneously. Midi and Shabbak (2011) studied the robust CUSUM control charting for the multivariate case. Lee (2011) proposed the economic design of the CUSUM chart for non-normally serially correlated data. S. F. Yang and Cheng (2011) proposed a new nonparametric CUSUM chart that is based on the sign test. They studied the ARL performance of the proposed chart for monitoring different location parameters. Similarly, much work has been done in the direction of robust EWMA control charting; for example, see Amin and Searcy (1991), S. F. Yang et al. (2011), and Graham et al. (2012).

Most of the CUSUM control charting techniques discussed above are based on first transforming the observed data into a nonparametric statistic and then applying the CUSUM chart on that transformed statistic. Unlike these approaches, L. Yang et al. (2010) proposed the use of a robust location estimator (i.e., the sample median) with the CUSUM control structure. Extending their approach, in this article we present a robust CUSUM chart that is based on five different estimators for monitoring the process location of phase II samples. The performance of the CUSUM chart with different robust estimators is studied in the presence of disturbances to normality, contaminations, outliers, and special causes in the process of interest. Before moving on toward the robust estimators, we provide the basic structure of the CUSUM chart in the next section.

THE CLASSICAL MEAN CUSUM CONTROL CHART

The mean CUSUM control chart proposed by Page (1954) has become one of the most popular methods to monitor processes. For a two-sided CUSUM chart, we plot the two statistics C_i^+ and C_i^- against a single control limit H. These two plotting statistics are defined as:

$$\left.\begin{array}{l} C_i^+ = \max[0, (\overline{X}_i - \mu_0) - K + C_{i-1}^+] \\ C_i^- = \max[0, -(\overline{X}_i - \mu_0) - K + C_{i-1}^-] \end{array}\right\}, \qquad [1]$$

where i is the subgroup number, $\overline{X}_i$ is the mean of study variable X, μ_0 is the target mean of the study variable X, and K is the reference value of the CUSUM scheme (cf. Montgomery 2009). The starting value for both plotting statistics is usually taken equal to zero; that is, $C_0^+ = C_0^- = 0$. Next, we plot these two statistics together with the control limit H. It is concluded that the process mean has moved upward if $C_i^+ > H$ for any value of i, whereas the process mean is said to be shifted downwards if $C_i^- > H$ for any value of i. Thus, the CUSUM chart is defined by two parameters—that is K and H—that have to be chosen carefully, because the statistical properties of the CUSUM chart are sensitive to these parameters. These two parameters are used in the standardized manner (cf. Montgomery 2009) and are given as:

$$K = k\sigma_{\overline{X}}, \quad H = h\sigma_{\overline{X}}, \qquad [2]$$

where $\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}}$, σ_X is the standard deviation of the study variable X and n is the sample size. The CUSUM statistic in [1] is given for the mean of samples.

DESCRIPTION OF THE PROPOSED ESTIMATORS AND THEIR CORRESPONDING CUSUM CHARTS

Let θ be the population location parameter that will be monitored through control charting and $\hat{\theta}$ be its estimator based on a subgroup of size n. There are many choices for $\hat{\theta}$ out of which we consider here the following: mean, median, mid-range, Hodges-Lehmann estimator, and trimean.

Based on a random sample $X_1, X_2, \ldots, X_n$ of size n, these estimators are defined as follows:

- **Mean:** This estimator is included in the study to provide a basis for comparison, because it is the most efficient estimator for normally distributed data. The sample mean $\overline{X}$ is defined as $\overline{X} = \frac{\sum_{j=1}^{n} X_j}{n}$, is a linear function of data, and is widely used to estimate the population location parameter. According to Rousseeuw (1991), the sample mean $\overline{X}$ performs well under the normality assumption but it is highly affected due to non-normality, is sensitive to outliers, and has zero breakdown point, which means that even a single inconsistent observation can change its value.
- **Median:** The sample median $\tilde{X}$ is defined as the middle-order statistic for odd sample sizes and the average of the two middle-order statistics in case of even sample sizes. The median is a robust estimator, so it is less affected by non-normality. Dixon and Massey (1969) showed that the efficiency of the median with respect to the mean decreases with an increase in sample size and the efficiency approaches 0.637 for large sample sizes.
- **Mid-range:** The mid-range is defined as $MR = \frac{X_{(1)} + X_{(n)}}{2}$, where $X_{(1)}$ and $X_{(n)}$ are the lowest and highest order statistics in a random sample of size n. It is highly sensitive to outliers because its design structure is based on only extreme values of data and therefore it has a zero breakdown point. Although its use is not very common due to its low efficiency for mesokurtic distributions, its design structure has the ability to perform good in the case of small samples from platykurtic distributions. Ferrell (1953) presented a comparison of both the efficiency of estimation and the detection of disturbances by medians and midranges. He indicated an advantage of these less conventional measures in detecting outliers while not losing much efficiency in detecting a change in the central value.
- **Hodges-Lehmann estimator:** This estimator is defined as the median of the pairwise Walsh averages; that is, $HL = \text{median}((X_j + X_k)/2,\ 1 \leq j \leq k \leq n)$. The main advantage of the HL estimator is that it is robust against outliers in a sample. It has a breakdown point of 0.29 (i.e., 29% is the least portion of data contamination needed to derive the estimate beyond all bounds; cf. Hettmansperger and McKean 1998). If the underlying distribution for the data is normal, then the asymptotic relative efficiency (*ARE*) of the *HL* estimator relative to the sample mean is 0.955; otherwise, it is often greater than unity (cf. Lehmann 1983).

- **Trimean:** The TriMean (*TM*) of a sample is the weighted average of the sample median and two quartiles: $TM = \frac{Q_1+2Q_2+Q_3}{4}$ (cf. Tukey 1977; Wang et al. 2007). It also equals the average of the median and the mid-hinge: $TM = \frac{1}{2}\left(Q_2 + \frac{Q_1+Q_3}{2}\right)$, where Q_p is the *p*th quartile of the sample (cf. Weisberg 1992). Like the median and the mid-hinge, but unlike the sample mean, *TM* is a statistically resistant L-estimator (a linear combination of order statistics) having a breakdown point of 25%. According to Tukey (1977), using *TM* instead of the median gives a more useful assessment of the location parameter. According to Weisberg (1992), the "statistical resistance" benefit of *TM* as a measure of the center (of a distribution) is that it combines the median's emphasis on center values with the mid-hinge's attention to the extremes. If the underlying distribution for the data is normal, then the *ARE* of the *TM* relative to the sample mean is 0.83 (cf. Wang et al. 2007).

The performance of these five estimators for phase I analysis could be seen in Schoonhoven et al. (2011a). In the current article, we study the performance of these estimators with the CUSUM structure for the phase II analysis. Following [1], the $\widehat{\theta}$-CUSUM control chart statistics S_i^+ and S_i^- can be represented as:

$$\left.\begin{aligned} S_i^+ &= \max\left[0, \left(\widehat{\theta}_i - \mu_0\right) - K_{\widehat{\theta}} + C_{i-1}^+\right] \\ S_i^- &= \max\left[0, -\left(\widehat{\theta}_i - \mu_0\right) - K_{\widehat{\theta}} + C_{i-1}^-\right] \end{aligned}\right\} \quad [3]$$

In [3], $\hat{\theta}$ may be any choice of the estimators mentioned above. Initial values for the statistics given in [3] are taken equal to zero; that is, $S_0^+ = S_0^- = 0$. The decision rule for the proposed chart(s), following the spirit of [2], is given in the form of $K_{\hat{\theta}}$ and $H_{\hat{\theta}}$ defined as:

$$K_{\hat{\theta}} = k_{\hat{\theta}}\sigma_{\widehat{\theta}}, \quad H_{\hat{\theta}} = h_{\hat{\theta}}\sigma_{\widehat{\theta}} \quad [4]$$

PERFORMANCE OF THE PROPOSED CUSUM CONTROL CHARTS

The performance of the proposed CUSUM control charting design structures given in [3] and [4] is evaluated in this section. The performance is measured in terms of their ARLs, which is the expected number of subgroups before a shift is detected. The in-control ARL is denoted by ARL_0 and an out-of-control ARL is denoted by ARL_1.

A code developed in R language is used to simulate ARL. For the said purpose, first we have simulated the standard errors for all estimators; that is, 5×10^5 samples of size n are generated and for each sample the value of estimator is determined. The standard deviation of those 5×10^5 values of estimator is represented by $\sigma_{\widehat{\theta}}$. Then the phase II samples of size n from different environment (explained later in this section) are generated under an in-control situation and the proposed charts are applied on those samples until an out-of-control signal is detected. The respective sample number (when the shift is detected by the chart) is noted, which is the in-control run length. This process is repeated 10,000 times so that we can average out these run lengths to get the ARL_0. Finally, a shift in the form of δ is introduced in the process to evaluate the ARL_1 performance of the proposed charts, where δ is the difference between μ_0 and μ_1, and μ_1 is the shifted mean. We have chosen to have $ARL_0 \cong 370$.

Uncontaminated Normal Environment

An uncontaminated normal distribution means that all observations come from $N\left(\mu_0, \sigma_0^2\right)$. The ARLs of the CUSUM charts using different estimators under the uncontaminated environment are given in Tables 1 and 2.

Tables 1 and 2 indicate that all estimators have almost the same in-control ARL; that is, $ARL_0 \cong 370$. For the out-of-control case, the ARLs of the CUSUM based on the mean are clearly smaller compared to the other estimators. Here the performance of the *HL* estimator is comparable to the mean estimator because the ARL performance of both is almost the same, whereas the other estimators have larger ARL_1 values. Moreover, the effect of change in subgroup size and $k_{\hat{\theta}}$ is identical for all estimators.

To further describe the run length distribution, we also report the standard deviations of the run length (SDRL) and some percentile points of the run length distribution in Tables 3 and 4, respectively.

Considering the results of Tables 3 and 4, we observe that the run length distribution of all of

TABLE 1 ARL Values for the CUSUM Chart Based on Different Estimators under Uncontaminated Normal Distribution with $k_{\hat{\theta}} = 0.25$ and $h_{\hat{\theta}} = 8.03$

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	370.096	24.592	10.006	6.353	4.645	3.151	2.395
	Median	372.498	31.594	12.377	7.702	5.600	3.734	2.850
	Mid-range	370.754	29.815	11.677	7.309	5.352	3.545	2.729
	HL	373.121	25.834	10.438	6.545	4.806	3.245	2.476
	TM	373.932	27.589	10.918	6.900	5.066	3.393	2.599
$n = 10$	Mean	372.356	15.352	6.722	4.391	3.322	2.251	1.976
	Median	372.746	18.954	8.053	5.194	3.877	2.653	2.057
	Mid-range	373.508	23.407	9.578	6.043	4.503	3.039	2.292
	HL	370.992	15.988	6.963	4.539	3.420	2.315	1.990
	TM	375.278	16.773	7.284	4.707	3.526	2.411	2.001

the proposed CUSUM charts is positively skewed as long as there is some variation in the run lengths. By decreasing the value of $k_{\hat{\theta}}$ from 0.5 to 0.25, the standard deviation of all of the proposed charts also decreases for small values of δ. The percentiles can also be used to compare the median run length (MRL) for CUSUM charts with different estimators.

Lucas and Crosier (1982) proposed the use of a fast initial response (FIR) feature with the CUSUM charts in which they recommended not to set the initial values of CUSUM statistics equal to zero. They found that the choice of a head start $S_0 = H_{\hat{\theta}}/2$ is optimal in the sense that its effect on the ARL_0 is very minor but it significantly decreases the ARL_1 values. The intention of FIR was to enhance the CUSUM chart sensitivity in detecting the shifts that occur immediately after the start of the process. As an example, we provide the ARL values of FIR CUSUM based on different estimators for $n = 10$, $k_{\hat{\theta}} = 0.5$, and $S_0 = H_{\hat{\theta}}/2$ at $ARL_0 \cong 370$. The results are given in Table 5, where the uncontaminated normal distribution is taken as the parent environment.

It is gratifying to note that the FIR feature indeed enhances the performance of the proposed CUSUM charts. In the upcoming sections we will not give the additional tables for the SDRL, percentile points of the run length, and the FIR features, although they can be easily obtained along the same lines.

Variance-Contaminated Normal Environment

A (φ) 100% variance-contaminated normal distribution is one that contains $(1-\varphi)$ 100% observations from $N\left(\mu_0, \sigma_0^2\right)$ and (φ) 100% observations from $N\left(\mu_0, \tau\sigma_0^2\right)$, where $0 < \tau < \infty$. The ARL values of the CUSUM charts using different estimators for the variance-contaminated normal environment are

TABLE 2 ARL Values for the CUSUM Chart Based on Different Estimators under Uncontaminated Normal Distribution with $k_{\hat{\theta}} = 0.5$ and $h_{\hat{\theta}} = 4.774$

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	370.469	28.338	8.304	4.788	3.383	2.242	1.803
	Median	374.278	41.831	11.267	6.073	4.205	2.665	2.065
	Mid-range	370.110	37.528	10.265	5.712	3.974	2.534	1.986
	HL	367.095	29.990	8.790	4.999	3.521	2.306	1.851
	TM	368.020	32.519	9.363	5.247	3.699	2.392	1.908
$n = 10$	Mean	367.292	14.845	5.156	3.177	2.364	1.708	1.148
	Median	367.663	19.538	6.390	3.837	2.774	1.946	1.468
	Mid-range	368.529	26.582	7.919	4.568	3.275	2.175	1.756
	HL	373.781	15.607	5.393	3.277	2.438	1.753	1.194
	TM	370.949	16.720	5.640	3.421	2.528	1.816	1.261

TABLE 3 SDRL Values for the CUSUM Chart Based on Different Estimators under Uncontaminated Normal Distribution with $n = 10$ at $ARL_0 \cong 370$

		δ						
Chart parameters	Estimator	0	0.25	0.5	0.75	1	1.5	2
$k_{\hat{\theta}} = 0.25$	Mean	360.523	6.590	1.902	0.994	0.641	0.437	0.164
$h_{\hat{\theta}} = 8.03$	Median	356.984	9.085	2.477	1.264	0.824	0.533	0.241
	Mid-range	360.781	12.457	3.195	1.611	1.021	0.564	0.459
	HL	350.709	6.980	1.991	1.032	0.669	0.469	0.142
	TM	356.536	7.674	2.143	1.099	0.706	0.501	0.130
$k_{\hat{\theta}} = 0.5$	Mean	367.992	9.498	1.963	0.939	0.577	0.465	0.355
$h_{\hat{\theta}} = 4.774$	Median	363.205	13.603	2.766	1.245	0.764	0.403	0.499
	Mid-range	361.144	20.159	3.764	1.623	0.975	0.479	0.446
	HL	367.477	10.194	2.141	0.988	0.626	0.450	0.395
	TM	365.022	11.056	2.278	1.043	0.647	0.421	0.439

given in Table 6 with $\phi = 0.05$ and $\tau = 9$ and in Table 7 with $\phi = 0.1$ and $\tau = 9$.

In Tables 6 and 7, the ARL_0 for the *TM* and *HL* CUSUM charts are affected least by the contamination, whereas the mid-range estimator is affected the most by this variance contamination. Similarly, in terms of ARL_1 values, *TM* and *HL* outperform all of the other estimators under discussion.

TABLE 4 Percentile Run Length Values for the CUSUM Chart Based on Different Estimators under Uncontaminated Normal Distribution with $n = 10$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ at $ARL_0 \cong 370$

		δ						
Estimator	Percentile	0	0.25	0.5	0.75	1	1.5	2
Mean	P_{10}	44	6	3	2	2	1	1
	P_{25}	111	8	4	3	2	1	1
	P_{50}	255	12	5	3	2	2	1
	P_{75}	504	19	6	4	3	2	1
	P_{90}	833.1	27	8	4	3	2	2
Median	P_{10}	45	7	3	2	2	1	1
	P_{25}	112	10	4	3	2	2	1
	P_{50}	256	16	6	4	3	2	1
	P_{75}	501	25	8	5	3	2	2
	P_{90}	843	37	10	5	4	2	2
Mid-range	P_{10}	46	8	4	3	2	2	1
	P_{25}	113	12	5	3	3	2	1
	P_{50}	255	21	7	4	3	2	2
	P_{75}	504	35	10	5	4	2	2
	P_{90}	828	53	13	7	4	3	2
HL	P_{10}	44	6	3	2	2	1	1
	P_{25}	111	9	4	3	2	1	1
	P_{50}	260	13	5	3	2	2	1
	P_{75}	518	20	7	4	3	2	1
	P_{90}	847.1	29	8	5	3	2	2
TM	P_{10}	44	6	3	2	2	1	1
	P_{25}	109	9	4	3	2	2	1
	P_{50}	258	14	5	3	2	2	1
	P_{75}	517	21	7	4	3	2	2
	P_{90}	846.1	31	9	5	3	2	2

Location-Contaminated Normal Environment

A (φ) 100% location-contaminated normal distribution is the one that contains $(1 - \varphi)$ 100% observations from $N(\mu_0, \sigma_0^2)$ and (φ) 100% observations from $N(\mu_0 + \omega\sigma_0, \sigma_0^2)$, where $-\infty < \omega < \infty$. The ARL values of the CUSUM chart using different estimators for a location-contaminated normal environment are given in Table 8 with $\phi = 0.05$ and $\omega = 4$.

Table 8 shows that none of the estimators is able to adequately detect the location contamination in the process when $n = 5$, because the ARL_0 for all the estimators is substantially lower than that for the uncontaminated environment. Increasing the subgroup size may be a better option because the mid-range and median CUSUM charts have a reasonable ARL_0 for $n = 10$ but the ARL_1 performance of the mid-range CUSUM chart is way too poor compared to the median CUSUM.

Special Cause Environment

Asymmetric variance disturbances are created in which each observation is drawn from $N(0,1)$ and has a φ probability of having a multiple of a $\chi^2_{(1)}$ variable added to it, with a multiplier equal to 4. ARLs of the CUSUM chart under this environment with $\phi = 0.01$ and $\phi = 0.05$ are given in Tables 9 and 10, respectively.

TABLE 5 ARL Values for the FIR CUSUM Chart Based on Different Estimators under Uncontaminated Normal Distribution with $n = 10$, $k_{\hat{\theta}} = 0.5$, and $S_0 = H_{\hat{\theta}}/2$ at $ARL_0 \cong 370$

		δ						
Estimator	$h_{\hat{\theta}}$	0	0.25	0.5	0.75	1	1.5	2
Mean	4.86	371.772	9.920	3.088	1.919	1.436	1.031	1.000
Median	4.87	371.999	14.114	3.886	2.297	1.694	1.134	1.007
Mid-range	4.9	370.930	19.588	4.853	2.699	1.969	1.295	1.043
HL	4.86	369.097	10.216	3.182	1.979	1.479	1.045	1.001
TM	4.86	372.281	11.417	3.364	2.069	1.523	1.068	1.002

In the presence of special causes, the median CUSUM seems more robust, whereas the mid-range CUSUM is affected the most. *TM* and *HL* CUSUM have good detection ability with a reasonable ARL_0. *TM* and *HL* estimators are affected positively by the increase in subgroup size; that is, their ARL_0 increase as we increase the value of n and vice versa.

Non-normal Environments

To investigate the effect of using non-normal distributions, we consider two cases: one by changing the kurtosis and the other by changing the symmetry of the distribution. For the case of disturbing the kurtosis, we use Student's t distribution with 4 degrees of freedom (T_4) and the logistic distribution (Logis(0,1)), and for the disturbance in symmetry we use the chi-square distribution with 5 degrees of freedom $\left(\chi^2_{(5)}\right)$. Tables 11–13 contain the ARL values for the proposed CUSUM charts under T_4, Logis(0,1), and $\chi^2_{(5)}$, respectively, where the ARL_0 is kept fixed at 370.

For the case of Student's t distribution, *TM* CUSUM performs the best among all of the other estimators followed by the *HL* CUSUM. Median CUSUM has also reasonable performance compared to the others, but the mid-range CUSUM seems to have worst performance for the said case (cf. Table 11). For the logistic distribution, the *HL* and *TM* CUSUM outperform the median and mid-range CUSUM charts, whereas the mean CUSUM reasonably maintains its performance (cf. Table 12). Similarly, *TM* and mean CUSUM charts show very good performance in case of chi-square distribution. *HL* also performs well, whereas the mid-range CUSUM has the worst performance (cf. Table 13).

We provide the ARL curves of the CUSUM charts with different estimators under different environments discussed above. Figures 1–5 contain the ARL curves of the CUSUM charts based on different estimators with $n = 10$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$.

From Figures 1–5 we see that the ARLs of the mid-range CUSUM are affected the most in case of some non-normal, contaminated, and special cause environments. The mean CUSUM is also poorly influenced under special cause environments. The ARL_0 values of the median, *TM*, and *HL* CUSUM

TABLE 6 ARL Values for the CUSUM Chart Based on Different Estimators under 5% Variance-Contaminated Normal Distribution with $k_{\hat{\theta}} = 0.5$ and $h_{\hat{\theta}} = 4.774$

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	303.375	40.319	10.911	5.900	4.129	2.630	2.037
	Median	353.227	44.057	11.895	6.454	4.424	2.789	2.125
	Mid-range	198.394	77.199	20.560	9.772	6.300	3.746	2.733
	HL	342.091	35.703	10.112	5.604	3.918	2.511	1.975
	TM	354.750	36.583	10.288	5.700	3.955	2.527	1.988
$n = 10$	Mean	330.826	20.222	6.475	3.844	2.803	1.959	1.474
	Median	366.873	21.549	6.797	4.009	2.891	1.995	1.545
	Mid-range	197.230	83.124	22.377	10.551	6.622	3.920	2.866
	HL	359.458	17.524	5.845	3.552	2.601	1.864	1.332
	TM	361.461	18.050	6.040	3.606	2.652	1.886	1.350

TABLE 7 **ARL Values for the CUSUM Chart Based on Different Estimators under 10% Variance-Contaminated Normal Distribution with $k_{\hat{\theta}} = 0.5$ and $h_{\hat{\theta}} = 4.774$**

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	296.883	51.619	13.442	7.054	4.816	2.984	2.226
	Median	340.203	49.462	12.875	6.927	4.704	2.924	2.222
	Mid-range	222.020	99.674	29.238	13.460	8.273	4.683	3.340
	HL	306.853	42.497	11.465	6.248	4.288	2.723	2.077
	TM	335.199	41.117	11.131	6.152	4.195	2.688	2.067
$n = 10$	Mean	329.818	25.775	7.771	4.482	3.196	2.141	1.727
	Median	356.416	22.899	7.161	4.213	3.026	2.057	1.624
	Mid-range	242.112	113.064	34.908	15.300	9.298	5.152	3.642
	HL	342.307	20.108	6.486	3.853	2.785	1.961	1.475
	TM	361.443	20.064	6.473	3.807	2.774	1.950	1.464

TABLE 8 **ARL Values for the CUSUM Chart Based on Different Estimators under 5% Location-Contaminated Normal Distribution with $k_{\hat{\theta}} = 0.5$ and $h_{\hat{\theta}} = 4.774$**

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	317.549	48.560	13.543	7.138	4.801	2.973	2.222
	Median	301.845	49.447	13.245	6.928	4.682	2.933	2.210
	Mid-range	315.737	78.997	25.153	12.361	7.818	4.429	3.124
	HL	285.025	45.634	12.369	6.565	4.485	2.806	2.124
	TM	297.391	43.218	11.775	6.330	4.372	2.742	2.085
$n = 10$	Mean	344.932	24.938	7.639	4.450	3.155	2.106	1.710
	Median	357.296	22.676	7.076	4.160	2.979	2.038	1.601
	Mid-range	391.235	78.931	24.038	11.693	7.523	4.367	3.097
	HL	303.981	21.554	6.811	3.978	2.879	1.986	1.543
	TM	329.528	20.369	6.464	3.869	2.793	1.957	1.489

seem less affected by the change of the parent normal environment.

For a graphical comparison of the proposed charts with non-normal environments, the ARL curves of all the charts under T_4, Logis(0,1), and $\chi^2_{(5)}$ distributions are given in Figures 6, 7, and 8, respectively, with $k_{\hat{\theta}} = 0.5$ and ARL_0 fixed at 370. These figures clearly indicate that, in general, *TM* and *HL* CUSUM are

TABLE 9 **ARL Values for the CUSUM Chart Based on Different Estimators under Special Cause Normal Distribution with $\varphi = 0.01$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$**

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	221.158	52.954	12.241	6.338	4.295	2.699	2.065
	Median	365.619	41.318	11.252	6.094	4.230	2.671	2.052
	Mid-range	168.113	132.188	48.782	15.514	8.731	4.740	3.301
	HL	356.893	31.754	9.094	5.156	3.607	2.359	1.888
	TM	364.754	33.872	9.704	5.368	3.762	2.418	1.921
$n = 10$	Mean	211.164	23.559	6.853	3.989	2.861	1.987	1.552
	Median	360.264	19.982	6.552	3.834	2.798	1.956	1.477
	Mid-range	124.994	104.625	75.651	27.395	12.790	6.089	4.115
	HL	367.158	15.936	5.460	3.345	2.465	1.784	1.215
	TM	375.058	16.858	5.690	3.464	2.546	1.834	1.281

TABLE 10 **ARL Values for the CUSUM Chart Based on Different Estimators under Special Cause Normal Distribution with $\varphi = 0.05$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$**

		δ						
Subgroup size	Estimator	0	0.25	0.5	0.75	1	1.5	2
$n = 5$	Mean	132.301	80.360	32.039	13.488	7.879	4.404	3.095
	Median	354.591	45.811	12.137	6.518	4.436	2.803	2.130
	Mid-range	104.606	90.670	76.205	58.787	38.613	14.055	7.940
	HL	292.820	45.662	11.647	6.180	4.230	2.680	2.056
	TM	322.227	38.766	10.660	5.794	4.076	2.573	2.008
$n = 10$	Mean	148.795	52.119	14.593	7.137	4.698	2.896	2.118
	Median	368.855	21.345	6.730	3.980	2.866	1.994	1.534
	Mid-range	105.884	91.084	75.093	62.220	48.336	24.478	12.724
	HL	345.449	18.332	6.031	3.625	2.645	1.878	1.357
	TM	355.257	18.601	6.020	3.639	2.656	1.889	1.366

TABLE 11 **ARL Values for the CUSUM Chart Based on Different Estimators under T_4 Distribution with $n = 10$ and $k_{\hat{\theta}} = 0.5$ at $ARL_0 \cong 370$**

		δ						
Estimator	$h_{\hat{\theta}}$	0	0.25	0.5	0.75	1	1.5	2
Mean	4.99	371.243	30.619	8.690	4.933	3.511	2.309	1.877
Median	4.83	371.753	24.568	7.443	4.366	3.123	2.110	1.682
Mid-range	5.80	369.540	205.442	53.968	20.771	12.092	6.505	4.501
HL	4.846	371.437	22.654	7.019	4.138	2.986	2.043	1.607
TM	4.84	370.038	22.075	6.877	4.055	2.932	2.024	1.581

TABLE 12 **ARL Values for the CUSUM Chart Based on Different Estimators under Standard Logistic Distribution with $n = 10$ and $k_{\hat{\theta}} = 0.5$ at $ARL_0 \cong 370$**

		δ						
Estimator	$h_{\hat{\theta}}$	0	0.25	0.5	0.75	1	1.5	2
Mean	4.790	371.335	47.114	12.424	6.671	4.541	2.860	2.166
Median	4.817	369.302	52.499	13.795	7.246	4.944	3.075	2.304
Mid-range	4.970	370.869	126.070	35.936	15.877	9.760	5.415	3.808
HL	4.817	368.761	45.081	12.161	6.498	4.479	2.819	2.139
TM	4.813	371.226	45.926	12.258	6.630	4.493	2.817	2.150

TABLE 13 **ARL Values for the CUSUM Chart Based on Different Estimators under Chi-square Distribution with $n = 10$ and $k_{\hat{\theta}} = 0.5$ at $ARL_0 \cong 370$**

		δ						
Estimator	$h_{\hat{\theta}}$	0	0.25	0.5	0.75	1	1.5	2
Mean	4.85	370.487	115.012	36.877	17.094	10.376	5.630	3.937
Median	4.90	369.970	127.209	44.319	20.670	12.017	6.527	4.454
Mid-range	5.19	369.765	196.527	96.916	49.213	28.913	13.261	8.395
HL	4.88	371.978	117.913	37.105	17.439	10.544	5.796	4.019
TM	4.87	370.087	115.104	37.648	17.085	10.494	5.760	4.000

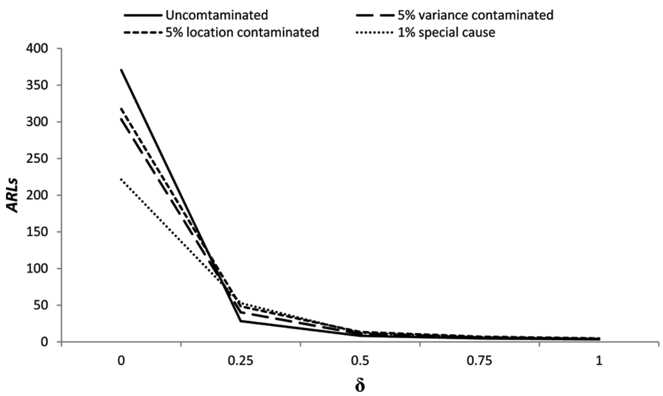

FIGURE 1 **ARL curves of the mean CUSUM with $n = 5$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ under different parent environments.**

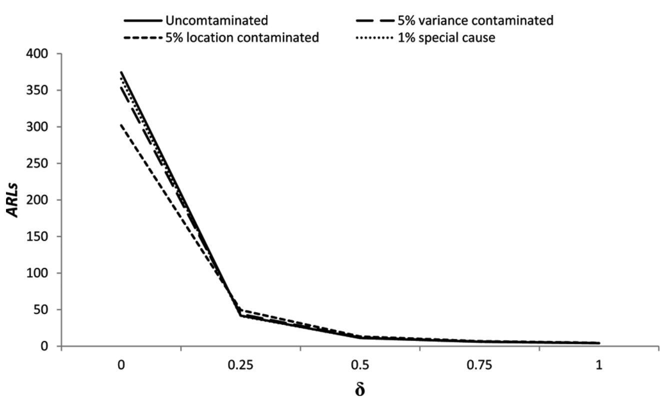

FIGURE 2 **ARL curves of the median CUSUM with $n = 5$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ under different parent environments.**

performing better than the other estimators under Student's *t* and logistic distributions. All of the estimators (except the mid-range) perform equally well in case of the chi-square parent environment.

Finally, we provide a comparison of our proposed CUSUM charts with the nonparametric CUSUM mean

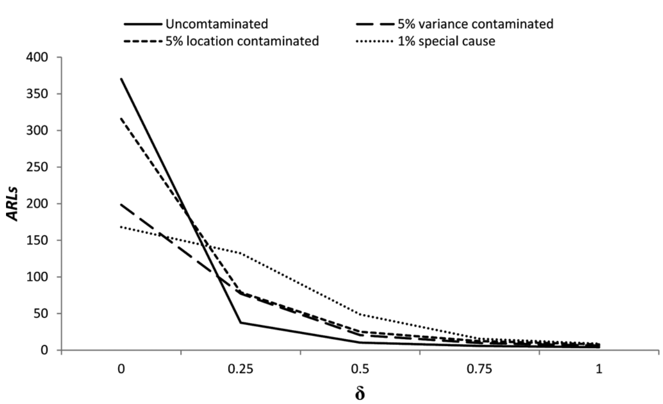

FIGURE 3 **ARL curves of the midrange CUSUM with $n = 5$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ under different parent environments.**

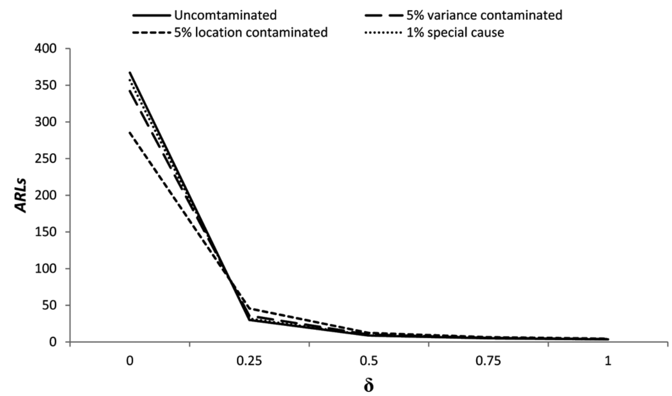

FIGURE 4 **ARL curves of the *HL* CUSUM with $n = 5$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ under different parent environments.**

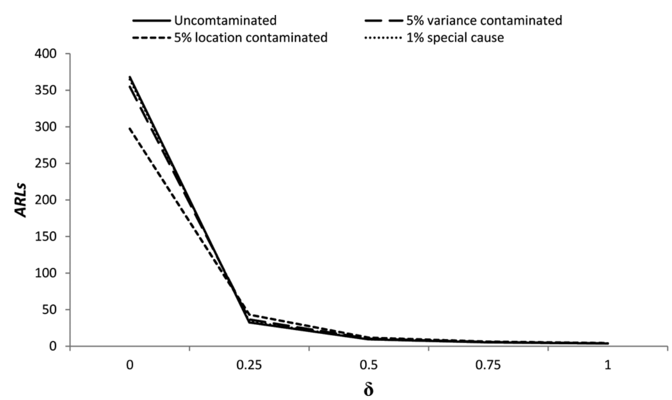

FIGURE 5 **ARL curves of the *TM* CUSUM with $n = 5$, $k_{\hat{\theta}} = 0.5$, and $h_{\hat{\theta}} = 4.774$ under different parent environments.**

chart (NPCUSUM) by S. F. Yang and Cheng (2011) under different environments discussed previously. S. F. Yang and Cheng (2011) calculated the ARLs of NPCUSUM using the shift parameter p_1. For a valid comparison of NPCUSUM with our proposed charts, we evaluated the ARL values of NPCUSUM chart

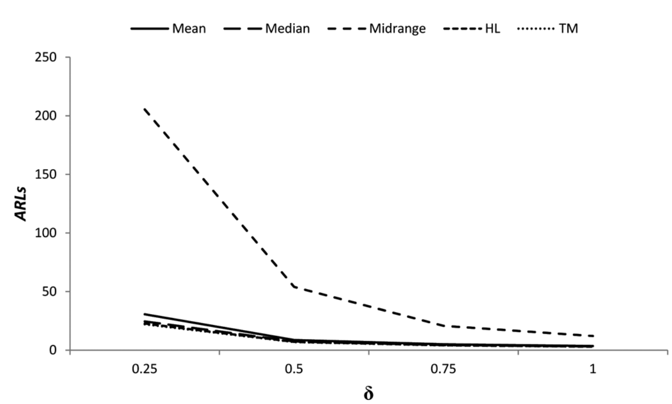

FIGURE 6 **ARL curves of different CUSUM charts under T_4 distribution with $n = 10$, $k_{\hat{\theta}} = 0.5$ and $ARL_0 \cong 370$.**

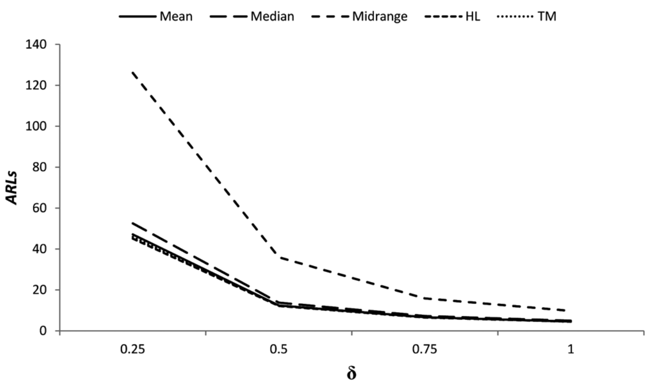

FIGURE 7 ARL curves of different CUSUM charts under logistic distribution with $n = 10$, $k_{\hat{\theta}} = 0.5$, and $ARL_0 \cong 370$.

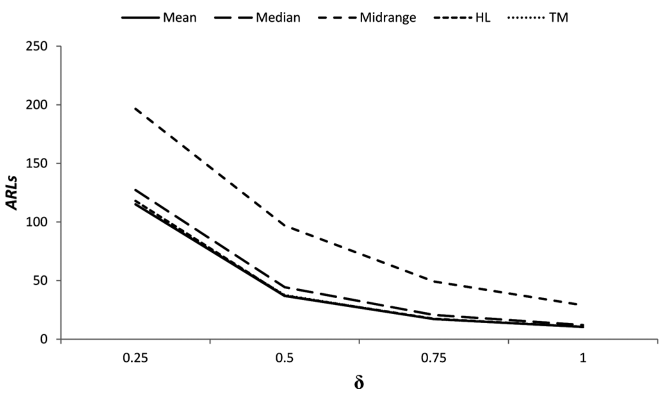

FIGURE 8 ARL curves of different CUSUM charts under chi-square distribution with $n = 10$, $k_{\hat{\theta}} = 0.5$, and $ARL_0 \cong 370$.

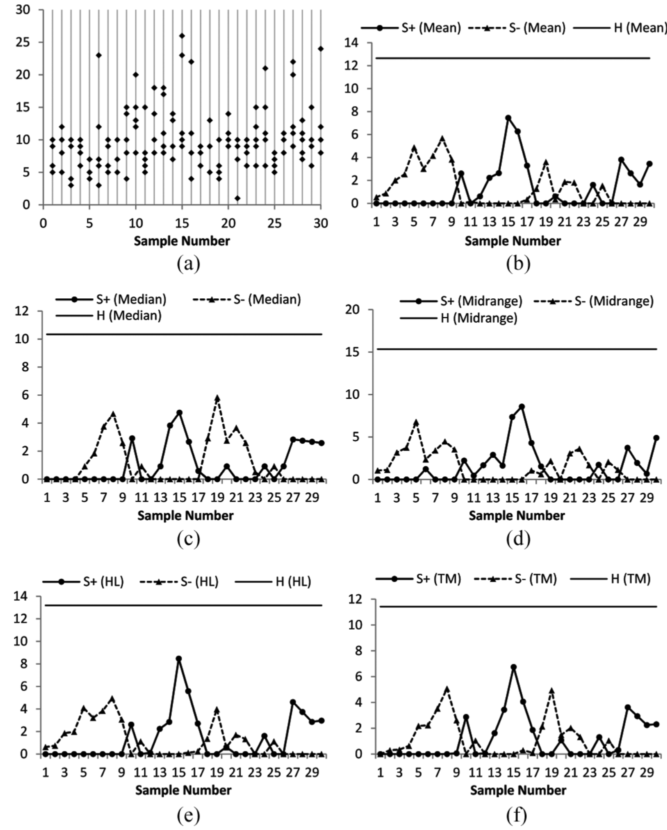

FIGURE 9 For uncontaminated data: (a) patients' waiting times; (b) output of mean CUSUM; (c) output of median CUSUM; (d) output of mid-range CUSUM; (e) output of *HL* CUSUM; and (f) output of *TM* CUSUM.

TABLE 14 ARL Comparison for the NPCUSUM (with $n = 10$, $k = 0.5$, and $h = 10.65$) and *TM* CUSUM (with $n = 10$ and $k_{\hat{\theta}} = 0.5$) Charts under Different Environments with Prefixed $ARL_0 = 370$

		δ						
Environment	Chart	0	0.25	0.5	0.75	1	1.5	2
Normal	NPCUSUM	371.051	20.492	8.266	5.465	4.274	3.330	3.030
	TM CUSUM	370.949	16.720	5.640	3.421	2.528	1.816	1.261
5% Variance contaminated	NPCUSUM	370.868	21.764	8.626	5.618	4.422	3.444	3.082
	TM CUSUM	361.461	18.050	6.040	3.606	2.652	1.886	1.350
10% Variance contaminated	NPCUSUM	370.739	22.915	9.037	5.866	4.571	3.548	3.156
	TM CUSUM	361.443	20.064	6.473	3.807	2.774	1.950	1.464
5% Location contaminated	NPCUSUM	172.225	15.595	7.558	5.205	4.162	3.289	3.025
	TM CUSUM	329.528	20.369	6.464	3.869	2.793	1.957	1.489
Special cause ($\varphi = 0.01$)	NPCUSUM	367.505	19.645	8.157	5.407	4.253	3.331	3.029
	TM CUSUM	375.058	16.858	5.690	3.464	2.546	1.834	1.281
Special cause ($\varphi = 0.05$)	NPCUSUM	243.051	16.922	7.716	5.244	4.183	3.307	3.029
	TM CUSUM	355.257	18.601	6.020	3.639	2.656	1.889	1.366
T_4	NPCUSUM	370.492	22.744	9.075	5.940	4.685	3.682	3.263
	TM CUSUM	370.038	22.075	6.877	4.055	2.932	2.024	1.581
Logis(0,1)	NPCUSUM	371.779	43.808	15.060	8.956	6.624	4.604	3.814
	TM CUSUM	371.226	45.926	12.258	6.630	4.493	2.817	2.150

using the shift parameter as δ, which is the difference between μ_0 and μ_1. The ARLs are calculated through Monte Carlo simulations by performing 10,000 replications of run lengths and are given in Table 14. For $\chi^2_{(5)}$, the conversion from p_1 to δ is not possible due to skewness of the distribution.

In some of the scenarios (such as location and 5% special cause contaminations), the NPCUSUM will perform better for small values of $\delta = 0.25$, but, in general, the *TM* (and *HL*) CUSUM have better performance across all the scenarios and for most values of δ.

A NUMERICAL EXAMPLE

In this section, we demonstrate the practical application of our proposed robust CUSUM control charting structures using a real data set about patients' waiting times (in minutes) for a colonoscopy procedure in a regional medical center (cf. Jones-Farmer et al. 2009). The data, consisting of 30 samples of size 5, are presented in Figure 9a. All of the proposed charts are applied on these data and the outputs of these charts are given in Figures 9b–9f. For the 10% variance contamination environment, we chose 10% observations randomly from the data and inflated their variance nine times; that is, $\tau = 9$. For 10% variance-contaminated data, the plot of data and the output of five charts is provided in Figure 10. Along similar lines, the location contamination and the outliers are introduced into the data and the respective data plots and chart outputs are given in Figures 11 and 12.

We can see that all of the proposed charts show the stability for uncontaminated distribution in Figure 9. For Figure 10, where the variance contamination is introduced into the process, the mid-range CUSUM detects a positive shift at sample 14, whereas the mean CUSUM detects the same shift at sample 15. The other three charts—that is, median, *HL*, and *TM* CUSUMs—were able to absorb the variance contamination without giving a false alarm. In case of location contamination, we can see that the plotting statistic (S^+) for all of the charts is inflated a bit, but no out-of-control signal is received for any chart (cf. Figure 11). Figure 12 is an interesting one

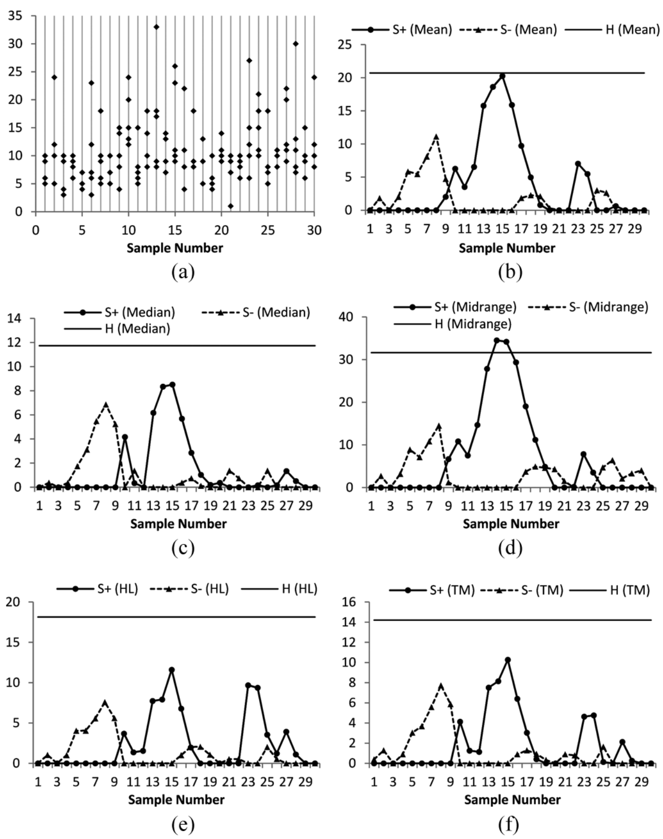

FIGURE 10 **For 10% variance-contaminated data: (a) patients' waiting times; (b) output of mean CUSUM; (c) output of median CUSUM; (d) output of mid-range CUSUM; (e) output of *HL* CUSUM; and (f) output of *TM* CUSUM.**

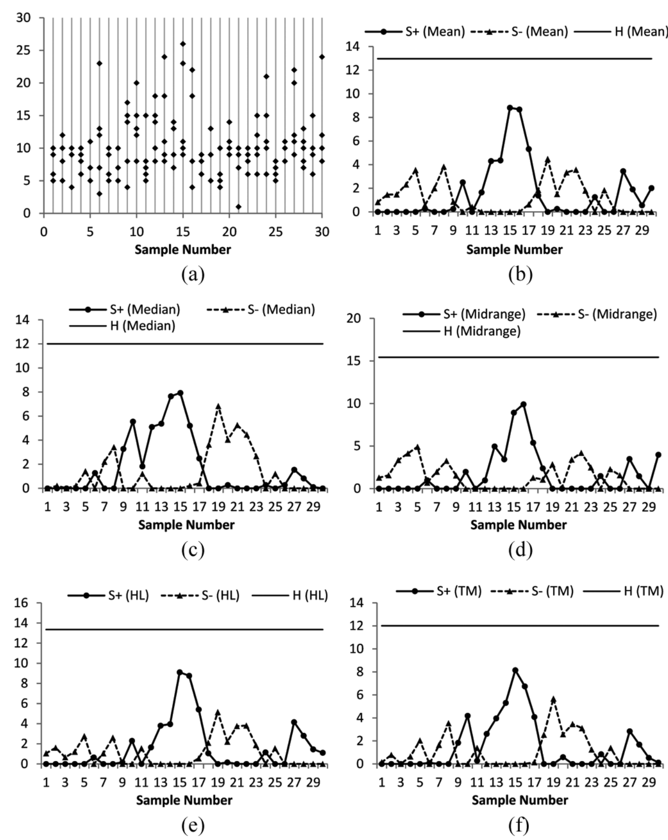

FIGURE 11 **For 5% location-contaminated data: (a) patients' waiting times; (b) output of mean CUSUM; (c) output of median CUSUM; (d) output of mid-range CUSUM; (e) output of *HL* CUSUM; and (f) output of *TM* CUSUM.**

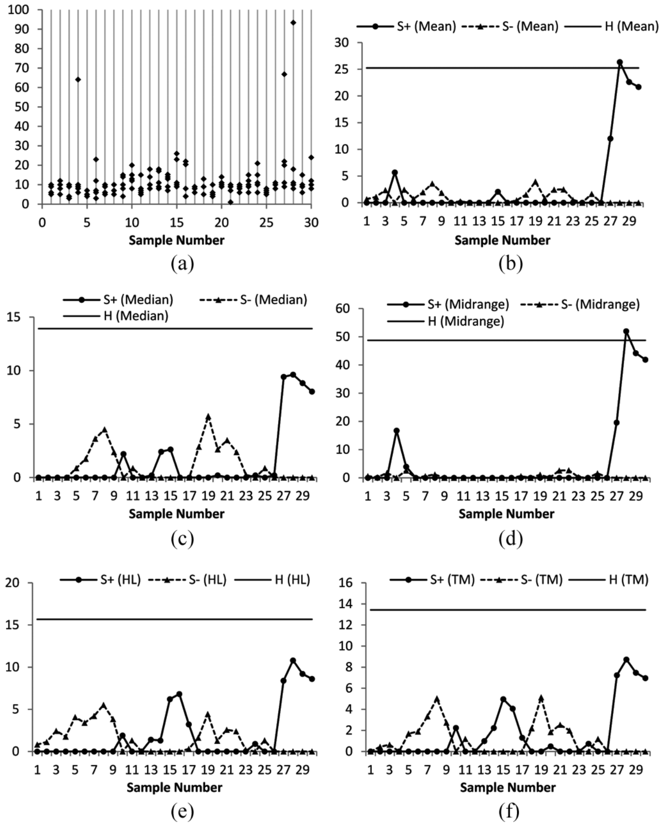

FIGURE 12 For special cause environment with $\varphi = 0.05$: (a) patients' waiting times; (b) output of mean CUSUM; (c) output of median CUSUM; (d) output of mid-range CUSUM; (e) output of *HL* CUSUM; and (f) output of *TM* CUSUM.

where we can see that there are three outliers present in the data set (cf. Figure 12a) at samples 4, 27, and 28. Median, *HL*, and *TM* CUSUMs assimilated these outliers, but the mean and mid-range CUSUM charts both gave the out-of-control signals at sample 27. The findings of this example are exactly on the same pattern as indicated by the ARL values for the proposed charts in Tables 1–13.

SUMMARY AND CONCLUSIONS

Common cause variations are an inherent part of any process, and with timely monitoring, evaluation, and identification of sources of special cause variations, the quality of the output of the process can be improved and waste (of time and cost) can be reduced significantly. Control charts are widely used to monitor a process. To monitor the location and dispersion parameters of the process, two main types of charts, Shewhart-type control charts and memory control charts (CUSUM and EWMA), are used. In practice, special cause variations and outliers may occasionally be present. The charts, which have a robust design structure, are used to cope with such environments. This study presents different robust design structures for CUSUM-type charts and evaluates their performance in different environments. The findings of the article are that the mean CUSUM control chart performs efficiently in uncontaminated environments and the *TM* and *HL* CUSUM charts are good compared to the mean CUSUM chart in this situation. The median CUSUM chart outperforms the other charts in the presence of special cause environment and outliers. The *TM* CUSUM chart is an alternative to the median chart and is highly efficient in the presence of outliers. It is also a good option in non-normal parent environments. Finally, the study concludes that the *TM* CUSUM chart is the best choice for controlling the location parameter of a process in normal, non-normal, special cause, and outlier environments.

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