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Robust CUSUM Control Charting for Process Dispersion

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Process monitoring through control charts is a quite popular practice in statistical process control. From a statistical point of view, a superior control chart is one that has an efficient design structure, but having resistance against unusual situations is of more practical importance. To have a compromise between the statistical and practical purposes, a natural desire is to have a control chart that can serve both purposes simultaneously in a good capacity. This study is planned for the same objective focusing on monitoring the dispersion parameter by using a Cumulative Sum (CUSUM) control chart scheme. We investigate the properties of the design structure of different control charts based on some already existing estimators as well as some new robust dispersion estimators. By evaluating the performance of these estimators-based CUSUM control charts in terms of average run length, we identify those charts that are more capable to make a good compromise between the aforementioned purposes in terms of statistical and practical needs.

Keywords: average run length (*ARL*); contamination; control chart; cumulative sum; robustness

1. Introduction

Existing control charts are usually designed under the assumptions of normality and outlier-free environments in the quality characteristic of concern. Normality seems more of a theoretical value, and it is generally hard to find practical situations where the normality assumption is easily fulfilled. Indeed, there are many practical situations where non-normality is more common.[1] On the basis of experience, it is common that processes have occasionally outliers in their outputs.[2] In case of violation of the normality assumption and the presence of outliers, these commonly used charts lose their efficiency and performance ability and hence are of less practical use. In general, robust control charts are preferred and are of more practical use when the design structure is not affected by the violation of aforementioned ideal assumptions.

The choice of the control charts to be used depends on the characteristics to be measured in the process and what type of amount of change/shift has to be determined. Control charts are classified into two categories, namely memoryless control charts and memory control charts. Shewhart-type control charts are termed as memoryless control charts, and their main deficiency is that they are less sensitive to small and moderate shifts in the parameters (location and dispersion). The commonly used memory control charts in the literature include Cumulative Sum (CUSUM) control charts[3] and Exponentially Weighted Moving Average (EWMA) control charts.[4] These memory control charts are designed such that they use the past information along with the current information, which makes them very sensitive to small and moderate shifts in the process parameters.

The control charting system is normally practiced in two distinct stages: Phase I (the retrospective phase) and Phase II (the prospective phase). In Phase I, the key concern is to understand the process and to access process stability, making sure that the process is operating at the intended target under some natural causes of variation. Phase I also involves the estimation of the parameters as well as setting up or estimating the control limits. In Phase II, the control chart is used to monitor the process on line to detect shifts occurring in the process so that any corrective actions can be taken quickly. Phase II focuses on the performance of the control, that is, how efficient the chart is to detect changes. Jensen *et al.*[5] suggested that more data in Phase I are needed than typically is recommended to achieve a performance comparable with the known parameters cases. In particular, for the CUSUM chart, the number of preliminary samples should be in the hundred scales rather than the dozen scales as used by the Shewhart chart.[6] For example, Quesenberry[7] recommended that at least 100 samples of size five should be used in Phase I for the CUSUM chart. That is because the CUSUM chart is sensitive to small shifts and any random error in the estimated parameter will tend to cause deviated in-control and out-of-control performance.

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In this paper, we concentrate on robust control charts for the process dispersion parameter in Phase II. For the related problem for the location parameter, the reader is referred to Schoonhoven *et al.*[8] for Shewhart-type charts, Nazir *et al.*[9] for the CUSUM charts, and Zwetsloot *et al.*[10] for the EWMA charts. For the dispersion parameter, the reader is referred to Schoonhoven *et al.*[11] for Shewhart-type charts.

Different authors have made contributions to apply robust estimators for monitoring the process dispersion parameter. The trimmed mean of subgroup ranges was proposed by Langenberg and Iglewicz[12] to estimate the process dispersion. The interquartile range (*IQR*) was proposed by Rocke[13] as an estimator. The modified bi-square A-estimator was recommended by Tatum.[14] De Mast and Roes[15] also used an A-estimator to construct the control limits for an individual control chart. Use of the mean of the subgroups medians absolute deviations (MAD) was suggested by Omar.[16] Abbasi and Miller[17] proposed a Phase II robust EWMA chart based on the mean absolute deviation from the median and showed that their proposed chart is efficient and robust against violation of the normality assumption under the restriction that a large clean data set is available in Phase I. They did not consider the presence of contaminations in Phase II. Schoonhoven *et al.*[11] studied the effectiveness and robustness of different dispersion estimators on Shewhart-type dispersion control charts in Phase I and Phase II under normality and cases where special causes are present.

The CUSUM control chart has received a great deal of attention in the quality control literature because of its simplicity and efficiency. It has been primarily used as a tool for monitoring process mean levels. The theoretical properties of the CUSUM chart for monitoring the process mean have been thoroughly investigated.[3,18,19] In contrast, the CUSUM chart as a tool for monitoring process variability has received less attention and investigation. Some published properties are found in Page,[20] Chang and Gan,[21] Hawkins and Olwell,[6] Acosta-Mejia,[22] Acosta-Mejia *et al.*,[23] and Acosta-Mejia and Pignatiello.[24] Note that the corresponding EWMA control chart is the subject of a PhD project at the University of Amsterdam.

In this paper, we compare a number of estimators that have been presented in the literature. Some of them are not common in the control charts literature. We derive the charts factors that determine the control limits. The performance of the charts based on these estimators is evaluated by assessing the average run length (*ARL*) under normality and in the presence of various types of contaminations by means of simulation.

Thus, the present paper focuses on robust CUSUM control charts for monitoring the process dispersion parameter. Particularly, their design structures and performances are studied under different parent environments and in the presence of special causes in the dispersion parameter of the process. The motivation and inspiration of this study is taken from Schoonhoven *et al.*[11] and Abbasi and Miller.[17] Before moving on towards the basic structure of the CUSUM chart, we provide a description of the robust estimators in the next section. Then we evaluate the performance of the different CUSUM charts by means of the *ARL*. Finally, we describe our main conclusions.

2. Description of estimators of process dispersion

Let θ be the process dispersion parameter, which needs to be monitored through control charting, and $\hat{\theta}$ be its estimator based on a sample of size n. There are many choices for $\hat{\theta}$. David[25] provided a brief history of standard deviation estimators. The traditional estimators are the pooled sample standard deviation, the mean of the sample standard deviations, and the mean of the sample ranges. Mahmoud *et al.*[26] studied the relative efficiencies of these estimators for different sample sizes n and number of samples k. Schoonhoven *et al.*[11] considered different estimators of the population standard deviation and provided a comprehensive analysis on their efficiency and use in control charts for different phases.

In deriving the estimates of the population dispersion parameter, we will look at some of the estimators discussed in Schoonhoven *et al.*[11] as well as some other robust estimators that are not common in the control charts literature. In the following, we give a short description of the estimators used in this study.

The first estimator of the population dispersion θ (which will also be used as a reference estimator throughout the rest of article) is the sample standard deviation S, which is defined by

$$S = \left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2\right)^{1/2}$$

where X_i denotes the ith observation in a sample of size n and $\overline{X}$ denotes the corresponding mean of the sample. The sample standard deviation S is the most efficient estimator in normally distributed environments, but studies have shown that it is highly affected by the presence of outliers and special causes. The breakdown point (proportion of the outlying observations that an estimator can cope) of the sample standard deviation is zero.

The next estimator is based on sample *IQR* and is defined as $IQR = \frac{Q_3 - Q_1}{1.34898}$, where Q_3 and Q_1 are the third and the first quartiles of the sample, respectively. Different properties of the sample *IQR* related to efficiency can be seen from Riaz.[27] The sample *IQR* is more robust to departures from normality and outliers than the sample standard deviation.[27] The breakdown point of *IQR* is 25%.

We also take into account an estimator proposed by Gini,[28] which is known as Gini's mean differences G and can be written as

$$G = \frac{4}{n-1}\sum_{i=1}^{n}\left(\frac{2i-n-1}{2n}\right)X_{(i)}$$

where $X_{(i)}$ is the ith order statistic of the sample. Gini's estimator is highly efficient and is more robust to outliers than the estimators based on the range and standard deviation.[29,30] Two similar estimators named as Downton's estimator $\left(D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left(i - \frac{n+1}{2}\right) X_{(i)}\right)$ and the probability-weighted moments-based estimator $\left(S_{pw} = \frac{\sqrt{\pi}}{n^2} \sum_{i=1}^{n} (2i - n - 1) X_{(i)}\right)$ are used by Khoo[31] and Muhammad and Riaz,[32] respectively, with the control structure of Shewhart's charts. The properties of those estimators are found to be similar to Gini's estimator because the three estimators are proportional to each other.

We also consider a robust estimator proposed by Hampel.[33] His robust estimator is based on the median of the absolute deviations from the median defined as

$$MADM = 1.4826 * \text{median}\left|X_i - \widetilde{X}\right|$$

where $\widetilde{X}$ is the sample median. This estimator is very robust against outliers, but its efficiency under normality is very low (i.e., only 37%). There are some more estimators based on the absolute deviations, that is, the mean of the absolute deviations from the mean $\left(MD = \frac{\sum_{i=1}^{n} \left|X_i - \overline{X}\right|/n}{1.2533}\right)$, the median of the absolute deviations from the mean $\left(MAD = \text{median}\left|X_i - \overline{X}\right|\right)$, and the mean of the absolute deviations from the median $\left(AADM = \sum_{i=1}^{n} \left|X_i - \widetilde{X}\right|/n\right)$. Wu *et al.*[34] showed that the *MADM* estimator performs the best, as compared with the other three estimators on the basis of absolute deviations, in case of contaminated environments. The breakdown point of *MADM* estimator is 50%. The gross error sensitivity (which measures the worst influence on the value of the estimator that a small amount of contamination of fixed size can have) of *MADM* is equal to 1.167, which is the smallest value that one can obtain for any scale estimator in the case of the normal distribution.

Rousseeuw and Croux[35] proposed different robust estimators of the population dispersion parameter θ, which are highly robust against outliers and their efficiencies under normality is higher compared with the estimator proposed by Hampel.[33] One of the estimators of Rousseeuw and Croux[35] is defined as

$$T_n = 1.38 * \frac{1}{h} \sum_{k=1}^{h} \{\text{median}|X_i - X_l|; i \neq l\}_{(k)}$$

This reads as follows: for each i, we compute the median of $|X_i - X_l|, l = 1, 2, \ldots, n$. This yields n values, the average of first h-order statistics gives the final estimate T_n, where $h = [n/2] + 1$, which is roughly half the number of observations (the symbol [·] represents the integer part of a fraction). Rousseeuw and Croux[35] also proposed two other robust estimators (similar to T_n), that is, $S_n = 1.1926 * \text{median}_i\{\text{median}_l |X_i - X_l|; i \neq l\}$ and $Q_n = 2.2219 * \{|X_i - X_l|; i < l\}_{(p)}$ with $p = \frac{h!}{2!(h-2)!}$. The breakdown point of the T_n estimator is 50%. T_n is very robust against outliers, and its efficiency under normality is 52%. The gross error sensitivity of T_n is 1.4688. The reason of choosing T_n is its low gross error sensitivity as compared with S_n and Q_n.

We also evaluate an estimator of the population dispersion parameter mentioned by Shamos[36] and Bickel and Lehmann.[37] This estimator is obtained by replacing pairwise averages by pairwise distances, and is defined as

$$B_n = 1.0483 * \text{median}\{|X_i - X_l|; i < l\}$$

This robust estimator has an efficiency of 86% under normality, but is less robust compared with the estimators proposed by Rousseeuw and Croux[35] and its breakdown point is only 29%.

The last estimator we use in this study is based on order statistics of certain subranges proposed by Croux and Rousseeuw[38] having a breakdown point of 50% and is defined as

$$S_r = 1.4826 * \left|X_{(i+[0.25n]+1)} - X_{(i)}\right|_{\left(\left[\frac{n}{2}\right] - [0.25n]\right)}$$

S_r is a very robust estimator in the presence of outliers, and its different features can be found in Croux and Rousseeuw.[38] Its efficiency under normality is only 37%; however, it is more efficient than *MADM* for small samples.

Further description and different properties (e.g., efficiency and robustness) of these estimators can be seen from Rousseeuw and Croux,[35] David,[25] Mahmoud *et al.*,[26] Schoonhoven *et al.*,[11] and Abbasi and Miller.[17]

It is anticipated that some robust estimators will perform well in an uncontaminated environment as well as in the presence of outliers and under special cause environments as the aim of these robust estimators is to estimate the population dispersion parameter efficiently and provide resistance against outliers and special cause environments.

Efficiency of the Estimators: For comparison purposes and to evaluate the accuracy of the dispersion estimators used in this study, we compute the standardized variances (*SV*) of the estimators as suggested by Rousseeuw and Croux[35] and relative efficiencies of the estimators as used by Abbasi and Miller.[17] The *SV* of a dispersion estimator $\hat{\theta}$ is calculated as

$$SV_{\hat{\theta}} = \frac{nVAR\left(\hat{\theta}\right)}{\left[E\left(\hat{\theta}\right)\right]^2}$$

The denominator of $SV\hat{\theta}$ is needed to obtain a natural measure of the accuracy of a scale estimator.[39] The relative efficiency (*RE*) of the estimator is computed as

$$RE_{\hat{\theta}} = \frac{\min\left(SV_{\hat{\theta}}\right)}{SV_{\hat{\theta}}}$$

SV and *RE* are computed by generating 10^5 samples of sizes $n = 4$, 5, and 9 under the following environments: uncontaminated normal, contaminated normal, gamma, Student's *t*, and logistic environments. The description of these environments is given in Section 4. *SV* and *RE* are given in Tables I and II, which read for example under uncontaminated normal environment as the dispersion estimator *S* as was expected has the lowest *SV* and the *MADM* has the largest SV. The efficiency of the other estimators falls within these two (*S* and *MADM*) estimators. Under 5% and 10% symmetric variance contaminated normal models, B_n has the lowest *SV* and for small 1% contamination, *G* obtains the lowest *SV*. For most of the non-normal environments, *G* and *IQR* are efficient estimators as compared with all other estimators. The dispersion estimator *S* is extremely affected by contaminations and non-normal environments.

3. The proposed CUSUM charts scheme for process dispersion

For the CUSUM procedures, we assume that we want to detect an increase in the process dispersion parameter θ. Let $\hat{\theta}$ be any estimator from Section 2 of the process dispersion parameter θ from a random sample of size *n*, which are taken from a continuous production process at regular intervals. The rule for the CUSUM-$\hat{\theta}$ charts is as follows:

$$Z_t = \max\left[0, \ \left(\hat{\theta} - K_{\hat{\theta}}\right) + Z_{t-1}\right], \qquad t = 1, 2, 3, \ldots$$

where $Z_0 = 0$ according to Tuprah and Ncube[40] and $K\hat{\theta}$ is the reference value for the scheme. Z_t is plotted against the sample number *t*. If $Z_t > H\hat{\theta}$ (where $H\hat{\theta}$ determines the decision interval) for any value of *t*, the process is deemed to be out of control and it is concluded that the process dispersion has increased. The sample number at which $Z_t > H\hat{\theta}$ is the run length of the process, and the expected value of the random variable run length is the *ARL* of the scheme. The values of $K\hat{\theta}$ are chosen in such a way that a shift in the process dispersion parameter is detected quickly. The values of $H\hat{\theta}$ are chosen for a fixed value of *ARL* along with value of $K\hat{\theta}$, when the process is in control under all environments considered in this study, and it is expected that the *ARL* will be small, when the process is out of control. The reference value $K\hat{\theta}$ will be based on Ewan and Kemp,[41] Page,[20] and Tuprah and Ncube,[40] so the value $K\hat{\theta}$ is taken to be half of expected values of $\hat{\theta}$ given $\theta_0 = 1$ and expected values of $\hat{\theta}$ given $\theta_1 = 1.4$, where θ_0 is the target value and θ_1 is the value of

Table I. Standardized variance of dispersion estimators under different environments

Environments	Sample size	Estimators						
		G	*IQR*	*S*	*MADM*	B_n	T_n	S_r
N(0,1)	4	0.7250	0.7250	0.7171	1.3092	0.7724	1.2804	1.3092
	5	0.6702	0.7028	0.6598	1.7131	0.8956	1.0250	1.6461
	9	0.5902	0.9259	0.5799	1.5378	0.7408	1.0647	1.3332
1%*CNormal*	4	0.8257	0.8257	0.8372	1.3358	0.8640	1.3077	1.3358
	5	0.7771	0.7857	0.8007	1.7377	0.9407	1.0575	1.6749
	9	0.6902	0.9308	0.7521	1.5468	0.7681	1.0791	1.3491
5%*CNormal*	4	1.1174	1.1174	1.1784	1.4268	1.1292	1.3998	1.4268
	5	1.0659	1.0154	1.1651	1.8343	1.0834	1.1510	1.7882
	9	0.9958	1.0024	1.2246	1.5910	0.8961	1.1503	1.4190
10%*CNormal*	4	1.3096	1.3096	1.3876	1.5758	1.3099	1.5500	1.5758
	5	1.2622	1.1855	1.3872	1.9584	1.2583	1.2867	1.9254
	9	1.1986	1.1338	1.4710	1.6683	1.0544	1.2391	1.5196
G(1,1)	4	1.5654	1.5654	1.6818	2.2602	1.6045	2.3068	2.2602
	5	1.5031	1.4771	1.6719	2.6164	1.8703	1.8482	2.5363
	9	1.4142	1.7224	1.7064	2.3855	1.6212	1.9844	2.2373
T_4	4	1.3564	1.3564	1.4559	1.6076	1.3670	1.5731	1.6076
	5	1.2898	1.2133	1.4401	2.0256	1.2645	1.3157	1.9658
	9	1.2494	1.2102	1.6207	1.7941	1.1040	1.3254	1.6316
Logis(0,1)	4	0.8972	0.8972	0.9018	1.4320	0.9178	1.3880	1.4320
	5	0.8604	0.8636	0.8735	1.8761	1.0491	1.1583	1.8221
	9	0.7786	1.0588	0.8129	1.6788	0.9028	1.1995	1.4934

Table II. Relative efficiencies of dispersion estimators under different environments

Environments	Sample size	Estimators						
		G	*IQR*	*S*	*MADM*	B_n	T_n	S_r
N(0,1)	4	98.9165	98.9165	100.0000	54.7773	92.8511	56.0095	54.7773
	5	98.4566	93.8921	100.0000	38.5162	73.6775	64.3705	40.0835
	9	98.2451	62.6240	100.0000	37.7069	78.2786	54.4628	43.4951
1%*CNormal*	4	100.0000	100.0000	98.6331	61.8148	95.5718	63.1449	61.8148
	5	100.0000	98.9111	97.0528	44.7188	82.6053	73.4821	46.3963
	9	100.0000	74.1547	91.7771	44.6220	89.8658	63.9606	51.1607
5%*CNormal*	4	100.0000	100.0000	94.8265	78.3189	98.9554	79.8274	78.3189
	5	95.2590	100.0000	87.1455	55.3539	93.7169	88.2139	56.7802
	9	89.9808	89.3916	73.1731	56.3196	100.0000	77.8978	63.1453
10%*CNormal*	4	100.0000	100.0000	94.3793	83.1068	99.9806	84.4896	83.1068
	5	93.9279	100.0000	85.4598	60.5369	94.2173	92.1382	61.5738
	9	87.9711	92.9959	71.6800	63.2022	100.0000	85.0965	69.3864
G(1,1)	4	100.0000	100.0000	93.0762	69.2590	97.5629	67.8590	69.2590
	5	98.2688	100.0000	88.3484	56.4567	78.9771	79.9242	58.2383
	9	100.0000	82.1044	82.8764	59.2821	87.2285	71.2664	63.2101
T_4	4	100.0000	100.0000	93.1645	84.3743	99.2279	86.2255	84.3743
	5	94.0692	100.0000	84.2510	59.8976	95.9523	92.2180	61.7203
	9	88.3615	91.2249	68.1185	61.5369	100.0000	83.2988	67.6655
Logis(0,1)	4	100.0000	100.0000	99.4937	62.6551	97.7565	64.6437	62.6551
	5	100.0000	99.6309	98.5057	45.8615	82.0180	74.2831	47.2222
	9	100.0000	73.5346	95.7754	46.3757	86.2379	64.9107	52.1343

process dispersion that needs to be detected quickly. Page[20] provided in his Table I, the reference values to notice a shift (that is $\theta_1 = 1.40$ to $\theta_1 = 2.23$) quickly in the process dispersion by using the sample range. Accordingly, $K\hat{\theta} = \left[E\left(\hat{\theta}|\theta_0\right) + E\left(\hat{\theta}|\theta_1\right)\right]/2$. As it is hard to find the analytical values of $E\left(\hat{\theta}|.\right)$, simulation is used for this purpose, and 50,000 random samples are generated from a normal distribution with mean ($\theta_0 = 1$ respectively $\theta_1 = 1.40$) and variance equal to 1, and the said expected value is evaluated. Table III gives the values of $K\hat{\theta}$ for samples of sizes $n = 5$ and $n = 9$ to detect a shift of size $\theta_1 = 1.4$.

For the values of $H\hat{\theta}$ under the different environments (normal and non-normal), we have searched out the values of $H\hat{\theta}$ by drawing random samples from the mentioned environments separately, and we have used an iterative method until the value of $H\hat{\theta}$ in each case is obtained that fixes an intended *ARL* along with reference value $K\hat{\theta}$. Values of $H\hat{\theta}$ with $ARL_0 = 500$ are given in Table IV. Alternative values of $H\hat{\theta}$ can be found in the same way for other choices of ARL_0.

The values of $K\hat{\theta}$ and $H\hat{\theta}$ have to be chosen carefully because the *ARL* performance of the CUSUM chart is sensitive to these values.

4. Performance evaluation of CUSUM-$\hat{\theta}$ charts

To assess the performance of the proposed CUSUM-$\hat{\theta}$ charts, the *ARL* is used as a performance measure. Monte Carlo simulation is used to determine the *ARL* of in-control and out-of-control processes. The simulation details are as follows: we have generated 10^5 random samples of size *n* from the parent environments (i.e., normal, contaminated normal or non-normal), and the concerned dispersion statistic (i.e., *S*, *IQR*,*G MADM*, B_n T_n, or S_r) is calculated. The corresponding control limits of the chart are developed using Tables III and IV. Then, the sample number at which the plotting statistic Z_t falls outside the control limits is noted. This noted sample number is called the run length, and it is a random variable. The same procedure is repeated 10^4 times to obtain the distribution of the run lengths. The mean of the run length distribution is represented by *ARL* and the standard deviation of run length distribution is represented by *SDRL*.

Table III. Values of $K\hat{\theta}$ for CUSUM-$\hat{\theta}$ charts under normal distribution

n	Estimator						
	S	*IQR*	*G*	*MADM*	B_n	T_n	S_r
5	1.13	1.47	1.35	0.98	1.32	1.35	1.15
9	1.16	1.34	1.35	1.09	1.26	1.25	1.20

Table IV. Values of $H\hat{\theta}$ for CUSUM-$\hat{\theta}$ charts for different environments with $ARL_0 = 500$

	N(0,1)		G(1,1)	T_4	*Logis*(0,1)
Estimator	$n = 5$	$n = 9$	$n = 5$	$n = 5$	$n = 5$
S	1.531	0.816	2.954	3.120	1.971
IQR	2.203	1.478	2.900	3.060	2.412
G	1.910	0.973	2.812	3.195	2.294
MADM	3.310	1.951	1.959	2.240	2.888
B_n	2.460	1.161	2.623	2.312	2.433
T_n	2.877	1.570	2.250	2.220	2.620
S_r	3.641	1.860	2.000	2.596	3.309

Inspired by Tatum[14] and Schoonhoven *et al*,[11] the performance of the CUSUM-$\hat{\theta}$ charts is evaluated under the following parent environments:

1. A model (say uncontaminated case) in which all observation are from *N*(0,1).
2. A model for symmetric variance disturbances in which each observation has 99% probability of being drawn from *N*(0,1) distribution and a 1% probability of being drawn from *N*(0,9).
3. A model for asymmetric variance disturbances in which each observation is drawn from a *N*(0,1) and has a 1% probability of having a multiple of a χ_1^2 variable added to it, with the multiplier equal to 4.
4. A model for mean disturbances in which each observation has a 99% probability of being drawn from *N*(0,1) distribution and a 1% probability of being drawn from the *N*(4,1) distribution.
5. To investigate the effect of using non-normal distributions, we consider two cases: one by disturbing the kurtosis and the other by disturbing the symmetry of the distribution. For the case of disturbing the kurtosis, we use Student's *t* distribution with four degrees of freedom (T_4) and the logistic distribution (*logis* (0,1)), and for the disturbance in symmetry, we use the gamma distribution (*G*(1,1)).

4.1. *Discussion of results*

The aforementioned environments are used, and in each case, the *ARL* values of the CUSUM-$\hat{\theta}$ charts are determined. We have considered shifts in terms of θ (i.e., $\delta\theta$), which means that the shifted dispersion parameter say θ' is defined as $\theta' = \delta\theta$. Here, $\delta = 1$ means no shift in θ, and the process dispersion is stable, and $\delta > 1$ means that the process θ has increased.

4.1.1. Uncontaminated case This environment is the basic assumption of the design structure of each chart. This provides a basis for comparisons for different control charts structures and hence for the proposed CUSUM-$\hat{\theta}$ charts. The following results can be observed from Table V.

Table V. *ARL* values of CUSUM-$\hat{\theta}$ charts under uncontaminated (*N*(0,1)) environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	*S*	501.95	74.11	40.19	25.07	17.93	7.16	4.80	3.84	2.57
	IQR	499.29	78.63	42.79	26.81	18.85	7.60	5.11	4.04	2.67
	G	500.26	75.62	40.72	25.11	18.27	7.30	4.88	3.92	2.61
	MADM	498.80	117.58	69.97	47.43	34.31	14.25	9.04	6.94	4.15
	B_n	498.34	90.86	49.86	32.39	22.43	8.97	5.95	4.67	2.97
	T_n	502.76	92.92	53.11	34.72	24.70	9.98	6.51	5.05	3.16
	S_r	498.13	111.97	69.05	45.60	33.42	13.52	8.81	6.67	3.97
9	*S*	500.54	54.86	26.61	16.04	11.13	4.58	3.26	2.71	2.10
	IQR	498.78	75.21	39.65	24.68	17.01	6.66	4.50	3.61	2.45
	G	499.58	56.07	27.43	16.30	11.36	4.66	3.30	2.75	2.11
	MADM	504.81	89.09	48.58	31.23	22.23	8.81	5.87	4.54	2.91
	B_n	503.25	62.87	32.31	19.37	13.32	5.42	3.75	3.07	2.22
	T_n	498.83	73.99	39.29	24.58	17.21	6.76	4.59	3.66	2.50
	S_r	503.40	79.68	43.75	27.74	19.34	7.81	5.20	4.14	2.72

When no contaminations are present, the CUSUM-$\hat{\theta}$ chart based on the sample standard deviation S performs the best as was to be expected, followed by the chart based on G. The IQR-based CUSUM chart works well as compared with charts based on B_n and T_n. The other CUSUM-$\hat{\theta}$ charts based on the remaining estimators ($MADM$ and S_r) are somewhat less efficient. Increasing the sample size from $n=5$ to $n=9$ results that the B_n and T_n based charts perform better as compared with IQR for $\delta < 1.25$, but for $\delta < 1.25$, the IQR chart works well (Table V).

To further explain the run length distribution under uncontaminated environment, we also report the $SDRL$ of the CUSUM-$\hat{\theta}$ charts to quantify the behavior of run length distribution as suggested by Antzoulakos and Rakitzis.[42] These are given in Table VI. When the process is in control, we want $SDRL$ to be close to its intended value, namely 500. Table VI reads that $SDRL$ is slightly lower to its intended value for some CUSUM-$\hat{\theta}$ charts and $SDRL$ decreases as δ increases for all charts.

Symmetric Variance Case: When symmetric disturbances are present, the best performing CUSUM charts are based on the $MADM$, S_r T_n followed by B_n. These estimators are very robust to outliers as these deviate less from the in-control ARL. The other CUSUM charts are very poor in the in-control situation as their ARLs deviate very much from the intended ARL (Table VII). The performance of the CUSUM charts based on S, G, and B_n become even worse when the sample size increases. However, the CUSUM chart based on IQR performs better than the one based on T_n for $n=9$.

Asymmetric Variance Case: When asymmetric variance disturbances are present, the CUSUM charts based on the estimators $MADM$, Sr, T_n, and B_n show good resistance against such disturbances and perform efficiently in detecting small shifts in the process dispersion parameters. The other charts perform very badly in maintaining the in-control ARL. The CUSUM chart based on IQR recovers quite substantially as the sample size becomes larger, but the performance of S and G is even worse (Table VIII).

Table VI. *SDRL* values of CUSUM-$\hat{\theta}$ charts under uncontaminated ($N(0,1)$) environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	S	489.16	70.57	35.38	20.73	13.32	3.69	1.98	1.38	0.69
	IQR	498.66	72.48	36.65	21.67	13.90	3.89	2.12	1.48	0.74
	G	483.77	69.34	35.19	20.21	13.49	3.73	2.03	1.41	0.71
	$MADM$	484.19	108.42	61.49	38.89	26.35	8.47	4.52	3.17	1.57
	B_n	488.85	84.24	43.78	26.34	16.83	4.88	2.61	1.81	0.90
	T_n	491.10	86.20	46.83	28.31	18.86	5.39	3.00	2.04	1.01
	S_r	489.99	102.42	60.96	37.09	25.50	8.01	4.49	3.08	1.49
	S	486.06	51.89	22.69	12.50	7.73	2.00	1.11	0.77	0.31
	IQR	495.56	65.46	32.31	18.19	11.44	3.12	1.66	1.14	0.55
	G	492.33	51.72	23.96	12.52	7.99	2.08	1.12	0.79	0.32
	$MADM$	495.82	82.93	42.48	25.65	16.82	4.78	2.61	1.77	0.88
	B_n	499.08	58.66	27.94	15.14	9.33	2.52	1.33	0.95	0.44
	T_n	494.86	68.97	34.22	19.75	12.45	3.40	1.85	1.25	0.64
	Sr	494.06	73.88	37.86	22.67	14.34	4.07	2.22	1.54	0.77

Table VII. *ARL* values of CUSUM-$\hat{\theta}$ charts under symmetric variance contaminated environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	S	123.33	43.29	28.40	19.98	14.82	6.76	4.67	3.77	2.55
	IQR	177.72	49.83	31.64	21.80	16.14	7.19	4.90	3.93	2.64
	G	145.58	45.37	29.23	20.42	15.28	6.86	4.72	3.83	2.59
	$MADM$	388.63	95.40	61.11	42.50	31.28	13.65	8.89	6.75	4.08
	B_n	291.45	67.63	40.72	27.95	20.55	8.65	5.72	4.57	2.93
	T_n	343.45	75.44	45.33	30.49	22.44	9.46	6.33	4.96	3.13
	S_r	362.72	93.62	58.73	40.65	29.99	12.97	8.47	6.55	3.92
	S	74.89	27.21	17.66	12.06	9.31	4.33	3.16	2.65	2.10
	IQR	355.39	58.00	31.95	20.41	14.78	6.01	4.16	3.36	2.35
	G	106.70	30.84	18.61	12.76	9.64	4.42	3.20	2.68	2.11
	$MADM$	392.09	76.71	43.75	28.73	20.62	8.52	5.65	4.48	2.89
	B_n	266.37	46.75	25.17	16.54	11.86	5.24	3.67	3.03	2.21
	T_n	342.77	59.86	33.90	21.96	16.11	6.59	4.51	3.62	2.48
	S_r	357.75	66.48	37.39	25.33	17.81	7.53	5.11	4.06	2.69

Table VIII. *ARL* values of CUSUM-$\hat{\theta}$ charts under asymmetric variance environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	S	65.00	37.27	26.11	19.57	14.86	6.80	4.70	3.77	2.58
	IQR	78.47	40.41	28.21	20.41	15.75	7.25	4.99	3.98	2.65
	G	70.16	38.21	26.66	19.85	15.13	6.91	4.80	3.86	2.59
	$MADM$	376.45	97.42	61.68	42.95	31.99	13.71	9.04	6.88	4.11
	B_n	251.93	65.73	40.43	27.31	20.29	8.70	5.82	4.58	2.97
	T_n	318.11	75.17	46.13	31.12	22.73	9.64	6.41	5.02	3.15
	S_r	359.13	95.05	60.27	41.39	30.77	13.22	8.60	6.68	3.94
	S	37.94	23.06	16.54	11.94	9.09	4.36	3.21	2.69	2.10
	IQR	328.43	57.56	32.12	20.86	14.80	6.10	4.21	3.40	2.37
	G	44.47	24.61	17.01	12.34	9.45	4.47	3.23	2.72	2.11
	$MADM$	390.99	78.62	44.70	29.54	21.35	8.61	5.74	4.52	2.90
	B_n	237.47	45.36	26.17	17.03	12.14	5.25	3.74	3.04	2.22
	T_n	337.68	61.14	34.64	22.36	16.18	6.63	4.54	3.63	2.49
	S_r	360.59	66.87	38.89	25.29	18.27	7.58	5.14	4.12	2.71

Location Contaminated Case: When disturbances are present in form of introducing outliers in the location of the process, then the CUSUM chart based on *MADM* performs well as it maintains more or less the in-control intended *ARL*, followed by charts based on S_r and T_n. All other charts perform very poor as they are unable to maintain the in-control properties to the target *ARL* (Table IX). Increasing the sample size results in an even better performance of the CUSUM chart based on *MADM*. The CUSUM chart based on *IQR* performs much better for a larger sample size, but the CUSUM charts based on *S*, *G*, and B_n deviate even more from their intended *ARL* in these circumstances.

Breakdown Points and Robustness of the Charts: Under an uncontaminated environment as was expected, no other CUSUM chart can perform better than the one based on *S*. It can be seen from Table VI that this chart outperforms all, followed by the CUSUM charts based on *G* and *IQR* for small samples and the CUSUM charts based on B_n and T_n work well for large samples. *MADM*-based CUSUM chart performs the worst from all as its efficiency under normality is very low (i.e., only 37%). But when there is contamination in the data, one can read from Tables VII and VIII that the *MADM*-based CUSUM chart (with 50% breakdown point and with low gross error sensitivity) maintains its in control properties well followed by the other robust estimators based charts S_r and T_n (both estimators have 50% breakdown points). With an increase in the sample sizes, the same output is observed but the in-control performance of *IQR*-based CUSUM chart (having 25% breakdown point) improves substantially. Under contaminations, the CUSUM charts based on *S* and *G* work poorly as both are based on non-robust estimators (e.g., *S* has a zero breakdown point).

Non-normal Cases: Without loss of generality, the drawn samples are transformed in such a way that the resulting sample has a mean equal to zero and a variance equal to one. For this purpose, the mean of concerned environment is subtracted from every drawn sample and then divided by the standard deviation of the concerned environment, so that valid and comparable results are evaluated. Under the gamma distribution, the CUSUM-$\hat{\theta}$ chart based on *IQR* performs best followed by the charts based on *S* and *G*.

Table IX. *ARL* values of CUSUM-v charts under location contaminated environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	S	64.86	31.68	23.03	17.24	13.61	6.56	4.65	3.75	2.56
	IQR	102.66	38.58	26.13	19.20	14.88	7.01	4.93	3.95	2.65
	G	80.54	34.50	23.79	17.60	14.06	6.61	4.73	3.80	2.60
	$MADM$	285.24	84.42	55.24	39.23	30.04	13.29	8.85	6.82	4.07
	B_n	177.21	53.92	34.92	24.65	18.74	8.26	5.68	4.55	2.93
	T_n	234.58	63.94	39.65	27.93	20.82	9.18	6.29	4.96	3.14
	S_r	267.55	79.41	52.26	36.86	28.22	12.73	8.47	6.53	3.94
	S	36.91	19.34	13.92	10.44	8.27	4.22	3.15	2.65	2.10
	IQR	234.50	49.35	28.35	18.56	13.72	5.90	4.12	3.34	2.36
	G	56.12	22.65	15.05	11.18	8.67	4.30	3.17	2.70	2.10
	$MADM$	325.30	68.19	40.11	26.75	19.68	8.44	5.61	4.46	2.89
	B_n	152.15	36.07	21.83	14.77	10.96	5.10	3.64	3.01	2.21
	T_n	252.78	52.25	30.34	20.34	14.79	6.43	4.45	3.59	2.48
	S_r	273.73	56.56	33.75	23.19	16.91	7.36	5.07	4.03	2.71

Table X. *ARL* values of CUSUM-$\hat{\theta}$ charts under *G*(1,1) environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	*S*	502.91	158.82	102.39	67.93	48.92	16.55	9.77	7.05	3.97
	IQR	501.02	157.33	97.27	64.05	46.22	14.96	8.54	6.30	3.58
	G	503.32	159.14	100.77	66.16	46.57	15.35	8.89	6.45	3.60
	MADM	504.29	219.76	152.19	110.23	81.42	27.41	14.49	9.54	4.69
	B_n	498.86	201.46	135.60	95.30	68.38	21.41	11.42	7.84	4.04
	T_n	504.79	196.02	127.77	90.87	64.85	21.05	11.10	7.51	3.84
	S_r	504.78	222.30	151.62	110.93	81.52	28.29	14.62	9.56	4.55

Table XI. *ARL* values of CUSUM- $\hat{\theta}$ charts under T_4 environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	*S*	499.31	209.97	135.56	87.64	60.88	17.61	9.95	7.18	3.94
	IQR	503.76	189.22	118.32	77.49	51.75	15.06	8.42	6.12	3.48
	G	505.82	194.44	122.35	80.49	54.45	15.9	8.91	6.49	3.65
	MADM	503.52	175.9	114.06	76.57	56.27	18.6	10.4	7.49	4.07
	B_n	500.6	172.75	111.75	72.34	49.35	14.52	8.12	5.89	3.35
	T_n	498.75	166.88	102.52	69.47	47.9	14.47	8.17	5.82	3.33
	S_r	500.47	169.8	109.51	74.32	53.85	17.73	10.31	7.32	4

Table XII. *ARL* values of CUSUM-$\hat{\theta}$ charts under *Logist*(0,1) environment when $ARL_0 = 500$

		δ								
n	Estimator	1	1.1	1.15	1.2	1.25	1.5	1.75	2	3
5	*S*	501.32	103.18	58.28	36.67	25.63	9.32	5.99	4.63	2.93
	IQR	500.76	100.59	56.41	35.88	25.02	9.12	5.85	4.54	2.87
	G	499.43	100.33	56.25	35.47	25.02	9.13	5.86	4.60	2.89
	MADM	506.61	137.01	83.99	56.96	41.17	15.38	9.47	7.07	4.09
	B_n	498.67	113.42	64.18	42.11	28.92	10.51	6.56	5.02	3.08
	T_n	498.65	117.24	66.95	43.99	30.27	11.04	6.90	5.22	3.21
	S_r	504.4	135.15	83.04	55.21	38.58	14.97	9.28	6.91	4.02

The charts based on the other estimators detect somewhat less efficiently shifts in the process (Table X). It can be observed from Table XI that the CUSUM-$\hat{\theta}$ charts based on T_n outperforms all other charts under the T_4 distribution followed by the chart based on S_r, B_n, and *MADM*. The CUSUM charts based on the *G* estimator performs efficiently followed by the charts based on *IQR* under the logistic distribution (Table XII). Other charts relatively work well under this environment.

5. Conclusion

In this article, we have considered several estimators of the dispersion parameter for the use in establishing phase II control limits. These estimators comprise some commonly used as well as robust estimators, which are not common in the control charts literature. A CUSUM chart scheme has been used to monitor the dispersion parameter using these estimators. The performance of these estimators has been assessed under various situations: the uncontaminated situation and various situations contaminated with symmetric and asymmetric variance disturbances, location disturbances, and non-normal environments. Under uncontaminated situation, all charts perform well, but the CUSUM chart based on the sample standard deviation *S* outperforms all, as was expected under normality. When there are symmetric and asymmetric variance disturbances, the CUSUM charts based on T_n, *MADM*, and S_r perform satisfactory and the performance of the other charts is (very) poor. The CUSUM chart based on the *IQR* estimator performs well for the gamma distribution, the *G* estimator performs well under the logistic distribution, and the T_n estimator performs well under the *t*-distribution. However, the differences between the estimators are not very substantial. In short, the dispersion CUSUM charts based on robust estimators (T_n, *MADM*, and S_r) behave well in all types of environments (uncontaminated, contaminated, and non-normal).

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